

# Evolutionary forms: Conservation laws and causality

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Evolutionary forms are skew-symmetric differential forms the basis of which, as opposed to exterior forms, are deforming manifolds (with unclosed metric forms). Such differential forms arise when describing physical processes.

A specific feature of evolutionary forms is the fact that from the evolutionary forms, which correspond to the conservation laws for material media, the closed exterior forms, which correspond to the conservation laws for physical fields, are obtained. This shows that material media generate physical fields. And by this the determinacy of physical processes and phenomena is revealed.

In this paper we obtain the mathematic apparatus that allows to describe discrete transitions and quantum jumps. This relates to the fact that the mathematic apparatus of exterior and evolutionary forms, which basis involves non-identical relations and degenerate transformations, *can describe transitions from nonconjugate operators to conjugate ones*. None of mathematic formalisms contains such possibilities.

The physical results that disclose a mechanism of evolutionary processes in material media and a generation of physical fields are obtained. These results explain many actual processes.

## Introduction

In the paper a role of skew-symmetric differential forms in mathematic physics and field theory is demonstrated.

It is known that the theories describing physical fields are based on the invariant methods of investigation (tensor, group, variational methods, theories of symmetries, transformations and so on) derived from the postulates.

Mathematical physics, which describes physical processes in material media, is based on the theory of differential equations.

In present paper it is shown that both mathematical physics and field theories are a whole. This is obtained on the basis of exterior and evolutionary skew-symmetric differential forms.

Closed exterior forms, which are differentials, have invariant properties. The invariant properties of exterior forms explicitly or implicitly manifest themselves essentially in all formalisms of field theory, such as the Hamilton formalism, tensor approaches, group methods, quantum mechanics equations, the Yang-Mills theory and others. They lie at the basis of field theory.

Evolutionary skew-symmetric differential forms are derived from differential equations describing physical processes in material media and forming the basis of mathematical physics.

In the paper it is shown that the closed exterior forms are obtained from evolutionary forms. This firstly disclose a relation between mathematical physics being based on differential equations and the invariant field theories. And sec-

ondly, this proves that material media generate physical fields. Thus it is disclosed a determinacy of physical processes and phenomena.

Such a role of exterior and evolutionary forms in physics relates to the fact that exterior and evolutionary forms reflect the properties of conservation laws, which, as it will be shown, play a controlling role in evolutionary processes.

In essence, in present paper two problems are solved.

Firstly, the mathematical apparatus, which contains nontraditional elements, namely, nonidentical relations and degenerate transformations, and allows to describe evolutionary processes and discrete transitions, has been obtained. Non of mathematical formalisms possesses such possibilities. Unique utilitarian possibilities of mathematical apparatus of skew-symmetric differential forms, which can serve as an approach to field theory, are shown.

And secondly, a mechanism of evolutionary processes in material media, which leads to generation of physical fields, is described.

One can judge about the content of present paper by the following list.

*1. Skew-symmetric differential forms.*

1.1 Some properties of manifolds. (The properties of manifolds are connected with a difference between exterior and evolutionary differential forms.)

1.2 Specific features of skew-symmetric differential forms.

1.3 Closed exterior differential forms. (Invariance, conjugacy, duality, and symmetries. Operators, identical relations and nondegenerate transformations).

1.4 Evolutionary skew-symmetric differential forms. (Nonclosure. Nonidentical relations and degenerate transformations. Selfvariation of nonidentical relation. Obtaining closed exterior forms from unclosed evolutionary forms. Transition from nonidentical relation to identical one under degenerate transformation. Relation between degenerate and nondegenerate transformations. Mechanism of realization of conjugated objects and operators.)

1.5 Evolutionary forms: Realization of differential-geometrical structures and forming pseudometric and metric manifolds. (Characteristics and classification of differential-geometrical structures realized.)

*2. Role of exterior forms in field theory.*

2.1 Conservation laws for physical fields (exact conservation laws).

2.2 Closed exterior differential forms in invariant field theories. (Exact conservation laws and specific features of existing field theories.)

*3. Role of evolutionary forms in mathematical physics and field theory.*

3.1 Balance conservation laws.

3.2 Evolutionary process in material medium and origination of physical structures. (Nonequilibrium state of material system. Selfvariation of nonequilibrium state of material system. Transition of material system into locally equilibrium state. Origination of physical structures.)

*4. Evolutionary forms: Properties of physical structures. Forming physical fields and manifolds.*

4.1 Characteristics of physical structures.

4.2 Relation between physical structures originated and material systems. (Potential forces.)

- 4.3 Characteristics of the formation created: intensity, spin, absolute and relative speeds of propagation of the formation.
- 4.4 Forming pseudometric and metric spaces. (Space of gravitational field).
- 4.5 Forming physical fields. Classification of physical structures.
- 5. *Conservation laws. Symmetries. Causality.*
- 5.1 Conservation laws. (Relation between conservation laws for material systems and conservation laws for physical fields.)
- 5.2 Symmetries.
- 5.3 Causality.
- 6. *Certain aspects of quantum field theory and approaches to general field theory.*
- 6.1 On interactions and classification of physical structures and physical fields.
- 6.2 Mathematical apparatus of skew-symmetric differential forms as the basis of the evolutionary field theory.

In *Appendix* presented at the end of the paper the analysis of balance conservation laws for thermodynamic and gas dynamic systems and for the system of charged particles is given.

## 1 Skew-symmetric differential forms

### 1.1 Some properties of manifolds.

A distinction of evolutionary skew-symmetric differential forms from exterior forms [1] is connected with the properties of manifolds on which skew-symmetric forms are defined.

It is known that manifolds with structures of any type can serve as a basis of exterior differential forms. They have one common property, namely, locally they admit one-to-one mapping into the Euclidean subspaces and into other manifolds or submanifolds of the same dimension [2].

While describing the evolutionary processes in material systems (material media) one is forced to deal with manifolds which do not allow one-to-one mapping described above. These can be manifolds constructed of trajectories of material system elements (particles). Such manifolds, which can be called accompanying manifolds, are deforming variable manifolds. In real processes such varying deforming manifolds cannot be manifolds of the types for which the theory of exterior differential forms has been developed.

The differential forms defined on these manifolds are evolutionary ones. The coefficients of these differential forms and the characteristics of accompanying manifolds are interconnected and are varied as functions of evolutionary variables.

A difference of manifolds on which exterior and evolutionary forms are defined relates to the properties of metric forms of these manifolds.

Assume that on the manifold one can set the coordinate system with base vectors  $\mathbf{e}_\mu$  and define the metric forms of manifold [3]:  $(\mathbf{e}_\mu \mathbf{e}_\nu)$ ,  $(\mathbf{e}_\mu dx^\mu)$ ,  $(d\mathbf{e}_\mu)$ .

The metric forms and their commutators define the metric and differential characteristics of the manifold.

If metric forms are closed (the commutators are equal to zero), the metric is defined  $g_{\mu\nu} = (\mathbf{e}_\mu \mathbf{e}_\nu)$ , and the results of translation over manifold of the point  $d\mathbf{M} = (\mathbf{e}_\mu dx^\mu)$  and of the unit frame  $d\mathbf{A} = (d\mathbf{e}_\mu)$  prove to be independent of the curve shape (the path of integration).

To describe the manifold differential characteristics and, correspondingly, the metric form commutators, one can use connectednesses [2-4]. If the components of metric form can be expressed in terms of connectedness  $\Gamma_{\mu\nu}^\rho$  [3], the expressions  $\Gamma_{\mu\nu}^\rho$ ,  $(\Gamma_{\mu\nu}^\rho - \Gamma_{\nu\mu}^\rho)$  and  $R_{\nu\rho\sigma}^\mu$  are components of the commutators of the metric forms with zeroth- first- and third degrees. (The commutator of the second degree metric form is written down in a more complex manner [3], and therefore it is not presented here).

The closed metric forms define the manifold structure. And the commutators of metric forms define the manifold differential characteristics that specify the manifold deformation: bending, torsion, rotation, and twist. (For example, the commutator of the zeroth degree metric form  $\Gamma_{\mu\nu}^\rho$  characterizes the bend, that of the first degree form  $(\Gamma_{\mu\nu}^\rho - \Gamma_{\nu\mu}^\rho)$  characterizes the torsion, the commutator of the third -degree metric form  $R_{\nu\rho\sigma}^\mu$  determines the curvature. (For manifolds with closed metric form of first degree the coefficients of connectedness are symmetric ones.)

Is is evident that the manifolds that are metric ones or possess the structure have closed metric forms. It is with such manifolds that the exterior differential forms are connected.

If the manifolds are deforming manifolds, this means that their metric form commutators are nonzero. That is, the metric forms of such manifolds turn out to be unclosed. The accompanying manifolds appearing to be deforming ones are the examples of such manifolds.

The skew-symmetric evolutionary differential forms whose basis are accompanying deforming manifolds are defined on manifolds with unclosed metric forms.

Below it will be shown that the specific properties of evolutionary skew-symmetric differential forms, which are defined on manifolds with unclosed metric forms, are connected with the properties of metric form commutators.

## 1.2 Specific features of skew-symmetric differential forms

The skew-symmetric differential form of degree  $p$  ( $p$ -form) can be written down as [4,5]

$$\omega^p = \sum_{\alpha_1 \dots \alpha_p} a_{\alpha_1 \dots \alpha_p} dx^{\alpha_1} \wedge dx^{\alpha_2} \wedge \dots \wedge dx^{\alpha_p} \quad 0 \leq p \leq n \quad (1.1)$$

where the local basis obeys the condition

$$\begin{aligned} dx^\alpha \wedge dx^\alpha &= 0 \\ dx^\alpha \wedge dx^\beta &= -dx^\beta \wedge dx^\alpha \quad \alpha \neq \beta \end{aligned}$$

The differential of skew-symmetric differential form can be written in the form

$$d\omega^p = \sum_{\alpha_1 \dots \alpha_p} da_{\alpha_1 \dots \alpha_p} dx^{\alpha_1} dx^{\alpha_2} \dots dx^{\alpha_p} + \sum_{\alpha_1 \dots \alpha_p} a_{\alpha_1 \dots \alpha_p} d(dx^{\alpha_1} dx^{\alpha_2} \dots dx^{\alpha_p}) \quad (1.2)$$

where the second term is connected with the differential of the basis.

[In further presentation a symbol of summing  $\sum$  and a symbol of exterior multiplication  $\wedge$  will be omitted. Summation over repeated indices is implied.]

The second term connected with the differential of the basis is expressed in terms of the metric form commutator. For differential forms defined on manifold with unclosed metric form one has  $d(dx^{\alpha_1} dx^{\alpha_2} \dots dx^{\alpha_p}) \neq 0$ . And for manifold with closed metric form it is valid the following  $d(dx^{\alpha_1} dx^{\alpha_2} \dots dx^{\alpha_p}) = 0$ .

That is, for differential forms defined on the manifold with unclosed metric form the second term is nonzero, whereas for differential forms defined on the manifold with closed metric form the second term vanishes.

For example, let us consider the first-degree form  $\omega = a_\alpha dx^\alpha$ . The differential of this form can be written as

$$d\omega = K_{\alpha\beta} dx^\alpha dx^\beta \quad (1.2')$$

where  $K_{\alpha\beta} = a_{\beta;\alpha} - a_{\alpha;\beta}$  are the components of the commutator of the form  $\omega$ , and  $a_{\beta;\alpha}$ ,  $a_{\alpha;\beta}$  are the covariant derivatives. If we express the covariant derivatives in terms of the connectedness (if it is possible), they can be written as  $a_{\beta;\alpha} = \partial a_\beta / \partial x^\alpha + \Gamma_{\beta\alpha}^\sigma a_\sigma$ , where the first term results from differentiating the form coefficients, and the second term results from differentiating the basis. (In the Euclidean space the covariant derivatives coincide with ordinary ones since in this case the derivatives of the basis vanish). If we substitute the expressions for covariant derivatives into the formula for commutator components, we obtain the following expression for the commutator components of the form  $\omega$

$$K_{\alpha\beta} = \left( \frac{\partial a_\beta}{\partial x^\alpha} - \frac{\partial a_\alpha}{\partial x^\beta} \right) + (\Gamma_{\beta\alpha}^\sigma - \Gamma_{\alpha\beta}^\sigma) a_\sigma \quad (1.3)$$

Here the expressions  $(\Gamma_{\beta\alpha}^\sigma - \Gamma_{\alpha\beta}^\sigma)$  entered into the second term are just the components of commutator of the first-degree metric form.

That is, the corresponding metric form commutator will enter into the differential form commutator.

If we substitute the expressions (1.3) for the skew-symmetric differential form commutator into formula (1.2'), we obtain the following expression for the differential of the first degree skew-symmetric form

$$d\omega = \left( \frac{\partial a_\beta}{\partial x^\alpha} - \frac{\partial a_\alpha}{\partial x^\beta} \right) dx^\alpha dx^\beta + ((\Gamma_{\beta\alpha}^\sigma - \Gamma_{\alpha\beta}^\sigma) a_\sigma) dx^\alpha dx^\beta$$

The second term in the expression for the differential of skew-symmetric form is connected with the differential of the manifold metric form, which is expressed in terms of the metric form commutator.

While deriving formula (1.3) for the differential form commutator the connectednesses of special type were used. However, a similar result can be obtained by applying the connectednesses of arbitrary type or by using another way for finding the differential of the base coordinates. For differential forms of any degree the metric form commutator of corresponding degree will be included into the commutator of the skew-symmetric differential form.

As it is known [4,5], the differential of exterior differential form involves only a single term. There is no second term. This indicates that the metric form commutator vanishes. In other words, the manifold, on which the *exterior* differential form is defined, has a closed metric form.

The differential of the evolutionary differential form, which is defined on the manifold with unclosed metric forms, will contain two terms: the first term depends on the differential form coefficients and the other depends on the differential characteristics of the manifold.

Thus, the differentials and, correspondingly, the commutators of exterior and evolutionary forms are of different types.

What does this lead to?

The commutator of *exterior* differential form contains only one term obtained from the derivatives of the differential form coefficients. Such a commutator may be equal to zero or nonzero depending on the form coefficients. This means that the exterior differential form may be closed or unclosed.

In contrast to this, the evolutionary differential form cannot be closed. The terms of the commutator of evolutionary differential form have a different nature. Such terms cannot compensate one another. For this reason, the evolutionary form commutator, and hence, the differential of evolutionary form, prove to be nonzero. And this means that the evolutionary form cannot be closed.

### 1.3 Closed exterior differential forms

#### Functional and utilitarian properties of closed exterior differential forms

Exterior differential form is called a closed one if its differential is equal to zero:

$$d\theta^p = 0 \quad (1.4)$$

From condition (1.4) one can see that the closed form is a conservative quantity. This means that it can correspond to the conservation law, namely, to some conservative physical quantity.

The differential of exterior form is a closed form. The exterior form which is the differential of some other form

$$\theta^p = d\theta^{p-1} \quad (1.5)$$

is called an exact form. (The exact forms prove to be closed automatically).

The closed inexact exterior forms possess utilitarian properties. These are exterior forms that are closed only on a certain structure (with the dimension being less than that of the initial manifold). In its metric form such a structure turns out to be a pseudostructure.

If the exterior form is closed only on pseudostructure, the closure condition is written as

$$d_\pi \theta^p = 0 \quad (1.6)$$

And the pseudostructure  $\pi$  obeys the condition

$$d_\pi^* \theta^p = 0 \quad (1.7)$$

where  $^* \theta^p$  is the dual form.

From conditions (1.6) and (1.7) one can see that the pseudostructure and the closed exterior form constitute a conservative object, namely, a quantity that is conservative on the pseudostructure. Hence, such an object can correspond to some conservation law. It is from such object that the physical fields are formed. (More detail description will be given below).

The exact form is, by definition, a differential. In this case the differential is a total one. The closed inexact form is a differential too. And in this case the differential is an interior (on pseudostructure) differential, that is

$$\theta_\pi^p = d_\pi \theta^{p-1} \quad (1.8)$$

Thus any closed form is a differential of the exterior form of a lower degree. From this it follows that the exterior form of lower degree may correspond to the potential, and the closed form by itself may correspond to the potential force. This is an additional example showing that the closed exterior form may have a physical sense. Here the two-fold nature of the closed form is revealed, on the one hand, as a conservative locally constant object, and, on the other hand, as a potential force.

Similarly to the differential connection between the exterior forms of sequential degrees, there is an integral connection. In particular, the integral theorems by Stokes and Gauss follow from the integral relation for  $p = 1, 2$  in three-dimensional space.

**Invariant properties of closed exterior differential forms.** Since the closed form is a differential (a total one if the form is exact, or an interior one on the pseudostructure if the form is inexact), it is obvious that the closed form turns out to be invariant under all transformations that conserve the differential. The unitary transformations (0-form), the tangent and canonical transformations (1-form), the gradient and gauge transformations (2-form) and so on are examples of such transformations. *These are gauge transformations for spinor, scalar, vector, tensor (3-form) fields.* It should be pointed out that just such transformations are used in field theory.

**Conjugacy and duality of exterior differential forms. Symmetries.** The closure of exterior differential form, which leads to invariance, is a result of

the conjugacy of elements of exterior or dual forms. The closure property of the exterior form means that any objects, namely, elements of the exterior form, components of elements, elements of the form differential, exterior and dual forms and others, turn out to be conjugated. It is a conjugacy that leads to realization of invariant and covariant properties of the exterior and dual forms.

Conjugacy is possible if there is one or other type of symmetry. Gauge symmetries, which are interior symmetries of field theory and with which gauge transformations are connected, are the symmetries of closed exterior differential forms. They are obtained as the result of conjugacy of any exterior form elements. The conservation laws *for physical fields* are connected with such interior symmetries.

(Below it will be shown that the exterior symmetries are symmetries of evolutionary differential forms, which are conditioned by degrees of freedom of *material systems*).

With the conjugacy another characteristic property of the exterior differential forms, namely, their duality, is connected. {The conjugacy is some identical connection between two operators or mathematical objects. The duality is a concept that means that one object carries a double meaning or that two objects with different meanings (of different physical nature) are identically connected. If one knows any dual object, one can obtain the other object}.

The connection of the exterior and dual forms is an example of the duality. The exterior form and the dual form correspond to the objects of different nature: the exterior form corresponds to the physical (i.e. algebraic) quantity, and the dual form corresponds to some spatial (or pseudospacial) structure. At the same time, under conjugacy the duality of these objects manifests itself, that is, if one form is known, it is possible to find the other form. In the case of closed inexact form the relation between the closed exterior form and the dual one leads to forming a differential-geometrical structure. (Physical structures that form physical fields are examples of such structures).

The duality is also revealed in the fact that, if the degree of the exterior form equals  $p$ , the dimension of the structure equals  $N - p$ , where  $N$  is the space dimension.

As the closed exterior form possesses invariant properties and the dual form corresponding to that possesses covariant properties, the invariance and covariance is one of examples of duality of the exterior differential forms.

The further example of the duality of the closed form is connected with the fact that, on the one hand, the closed exterior form is conservative quantity, and, on the other hand, the closed form can correspond to potential force. (Below the physical meaning of this duality will be elucidated, and it will be shown in respect to what the closed form manifests itself as potential force and with what the conservative physical quantity is connected).

As it will be shown below, the duality of closed differential forms has a fundamental physical meaning. The duality is a tool, which untangles the mutual connection, the mutual changeability and the transitions between different physical objects in evolutionary processes.



## Specific features of the mathematical apparatus of exterior differential forms.

**Operators of the theory of exterior differential forms.** In differential calculus the derivatives are basic elements of the mathematical apparatus. By contrast, the differential is an element of mathematical apparatus of the theory of exterior differential forms. It enables one to analyze the conjugacy of derivatives in various directions, which extends the potentialities of differential calculus.

The operator of exterior differential  $d$  is an abstract generalization of ordinary mathematical operations of the gradient, curl, and divergence in the vector calculus [5]. [Here it should be emphasized that usual concepts of gradient, curl and divergence are operators applied to vector, whereas a gradient, curl and divergence in the theory of exterior forms, which are obtained as a result of exterior differentiation, are operators applied to pseudovector (an axial vector).]

If, in addition to the exterior differential, we introduce the following operators: (1)  $\delta$  for transformations that convert the form of  $p + 1$  degree into the form of  $p$  degree, (2)  $\delta'$  for cotangent transformations, (3)  $\Delta$  for the  $d\delta - \delta d$  transformation, (4)  $\Delta'$  for the  $d\delta' - \delta'd$  transformation, one can write down the operators in the field theory equations in terms of these operators that act on the exterior differential forms. The operator  $\delta$  corresponds to Green's operator,  $\delta'$  does to the canonical transformation operator,  $\Delta$  does to the d'Alembert operator in 4-dimensional space, and  $\Delta'$  corresponds to the Laplace operator [6]. It can be seen that the operators of the exterior differential form theory are connected with many operators of mathematical physics.

The mathematical apparatus of exterior differential forms extends the potentialities of the integral calculus. As it has been pointed out, the exterior differential forms were introduced as integrand expressions for definition of the integral invariants. The closure condition for exterior differential forms makes it possible to find the integrability conditions. This fact is of great importance in applications.

**Identical relations of exterior differential forms.** The basis of the mathematical apparatus of closed exterior differential forms is formed by identical relations. (Below it will be shown that at the basis of the mathematical apparatus of evolutionary differential forms there lie nonidentical relations, and it will be shown that identical relations for exterior differential forms are obtained from nonidentical relations for evolutionary forms.)

The identical relations of exterior differential forms are an mathematical expression of various kinds of the conjugacy or the duality of closed differential forms. (Since the conjugacy is a certain connection between two operators or mathematical objects, it is evident that the relations can be used to express the conjugacy mathematically.) They describe the conjugacy of any objects: the form elements, components of each element, exterior and dual forms, exterior forms of various degrees, and others. The identical relations that are connected with different kinds of conjugacy elucidate invariant, structural and

group properties of exterior forms, which is of great importance in applications.

The identical relations for exterior differential forms follow from the closure conditions of differential forms, namely, vanishing the form differential (see formulae (1.4), (1.6), (1.7)) and the conditions connecting the forms of consequent degrees (see formulae (1.5), (1.8)).

The identical relation, which connects exterior forms of consequent degrees, express the fact that each closed exterior form is a differential of some exterior form. In general form such an identical relation can be written as

$$d_\pi \phi = \theta_\pi^p \quad (1.9)$$

In this relation the form in the right-hand side has to be a *closed* one. (As it will be shown below, the identical relations are satisfied only on pseudostructures).

In identical relation (1.9) in one side it stands a closed form and in other side does the differential of some differential form (the differential is a closed form as well). The examples of such identical relations are a) the Poincare invariant  $ds = -H dt + p_j dq_j$ , b) the second principle of thermodynamics  $dS = (dE + p dV)/T$ , d) the conditions on characteristics in the theory of differential equations and so on. The requirement that the function is an antiderivative (the integrand is a differential of a certain function) can be written in terms of such an identical relation. Existence of harmonic function is written by means of the identical relation: the harmonic function is a closed form, that is, a differential.

In addition to relations in differential forms, from the closure conditions of differential forms and the conditions connecting the forms of consequent degrees the identical relations of other types are obtained. This is also connected with the fact that the exterior forms can have the tensor, differential or integral representation. As the example of such relations it can serve

*integral identical relations* (formulae by Newton, Leibnitz and Green and integral relations by Stokes, Gauss-Ostrogradskii, etc);

*tensor identical relations* (vector and tensor identical relations that connect the operators of gradient, curl, divergence and so on);

*identical relations between derivatives* (the Cauchy-Riemann conditions in the theory of complex variables, the transversality condition in the calculus of variations, the canonical relations in the Hamilton formalism, the thermodynamic relations between derivatives of thermodynamic functions [7], the condition that the derivative of implicit function is subject to, the eikonal relations [8] and so on.

As the examples of identical relations it can serve such relations as the gauge relations in electromagnetic field theory, the tensor relations between connectednesses and their derivatives in gravitation (the symmetry of connectednesses with respect to subscripts, the Bianchi identity, the conditions imposed on the Christoffel symbols) and so on.

The identical relations of exterior differential forms appear practically in all branches of physics, mechanics and thermodynamics.

The physical meaning of identical relations for exterior differential forms will be disclosed below using evolutionary differential forms.

**Nondegenerate transformations.** One of the fundamental methods in the theory of exterior differential forms is an application of *nondegenerate* transformations. The nondegenerate transformations, if applied to identical relations, enable one to obtain new identical relations and new closed exterior differential forms.

Nondegenerate transformations in the theory of exterior differential forms are those that conserve the differential. As it has been pointed, the unitary, tangent, canonical, gradient, and gauge transformations are examples of such transformations.

From description of operators of exterior differential forms one can see that they are operators which execute some transformations. All these transformations are connected with the nondegenerate transformations of exterior differential forms.

The significance of the nondegenerate transformations consists in the fact that they allow to find closed differential forms.

As one can see, the properties of closed exterior differential forms reveal in many branches of physics.

In the second part of the paper it will be demonstrated a role of exterior differential forms in field theory. This role is connected with the fact that closed exterior forms reflect the properties of conservation laws for physical fields.

#### 1.4 Evolutionary skew-symmetric differential forms

The properties of exterior differential forms and specific features of their mathematical apparatus have a great utilitarian and functional importance. However, the question of how closed exterior differential forms are obtained and how the process of conjugating different objects is realized remains unsolved.

The mathematical apparatus of evolutionary differential forms enables us to answer this question.

#### Nonclosure of evolutionary differential forms.

It has been shown above that the exterior skew-symmetric differential forms can be closed. Such skew-symmetric differential forms possess invariant properties, and the basis of their mathematical apparatus includes identical relations and degenerate transformations. (Just on these properties the field theory is based.)

The evolutionary forms, as opposed to exterior forms, cannot be closed. This relates to the fact that the manifolds, on which the evolutionary forms are defined, have unclosed metric forms. The commutator of unclosed metric form, which is nonzero, enters into the evolutionary form commutator. Such commutator of differential form cannot vanish. This means that the evolutionary form differential is nonzero. Hence, the evolutionary differential form, in contrast to the case of the exterior form, cannot be closed and does not possess invariant properties.

As it will be shown below, the basis of mathematical apparatus of evolutionary differential forms, which are unclosed forms, includes *nonidentical* relations and *degenerate* transformations. (Nonidentical relations under degenerate transformations are just the relations that generate identical relations for exterior forms.)

## Nonidentical relations of evolutionary differential forms

The identical relations of closed exterior differential forms reflect a conjugacy or duality of any objects. The evolutionary forms, being unclosed, cannot directly describe the conjugacy of any objects. But they allow a description of *the process in which the conjugacy may appear* (that is, to describe how closed exterior differential forms are generated). Such a process is described by nonidentical relations.

The identical relations establish exact correspondence between the quantities (or objects) involved into this relation. It is possible in the case when the quantities involved into the relation are measurable (invariant) ones. In the nonidentical relations one of the quantities is unmeasurable.

[The concept of “nonidentical relation” may appear to be inconsistent. However, as it will be shown below, this has a deep meaning.]

The relation of evolutionary forms is a relation between the differential, which is a closed differential form and is a measurable (invariant) quantity, and the evolutionary form that is unclosed form and hence is not a measurable quantity. Such a relation cannot be identical.

Nonidentical relations of such type appear in descriptions of physical processes.

Nonidentical relations may be written as

$$d\psi = \omega^p \quad (1.10)$$

Here  $\omega^p$  is the  $p$ -degree evolutionary form, which is unclosed form,  $\psi$  is a certain form of degree  $(p - 1)$ , and the differential  $d\psi$  is a closed form of degree  $p$ .

In the left-hand side of this relation it stands the form differential, i.e. a closed form that is an invariant object. In the right-hand side it stands the unclosed form that is not an invariant object. Such a relation cannot be identical. {Nonidentical relation was analyzed in paper J.L.Synge “Tensorial Methods in Dynamics” (1936). And yet it was allowed a possibility to use the sign of equality in nonidentical relation.}

One can see a difference of relations for exterior forms and evolutionary ones. In the right-hand side of identical relation (1.9) it stands a closed form, whereas the form in the right-hand side of nonidentical relation (1.10) is an unclosed one.

Relation (1.10) is an evolutionary relation as it involves the evolutionary form.

Such nonidentical relations appear, for example, while investigating the integrability of any differential equations that describe various processes.

As the example one can inspect the partial differential equation of the first order. Let

$$F(x^i, u, p_i) = 0, \quad p_i = \partial u / \partial x^i \quad (1.11)$$

be a partial differential equation of the first order.

Let us consider the functional relation

$$du = \theta \quad (1.12)$$

where  $\theta = p_i dx^i$ . Here  $\theta = p_i dx^i$  is the differential form of the first degree (summation over repeated indices is implied).

In the general case, for example, when differential equation (1.11) describes any physical processes, functional relation (1.12) turns out to be nonidentical.

For this relation to be identical, the differential form  $\theta = p_i dx^i$  must also be a differential (like the left-hand side of relation (1.12)), that is, it has to be a closed exterior differential form. To do this, it requires the commutator  $K_{ij} = \partial p_j / \partial x^i - \partial p_i / \partial x^j$  of the differential form  $\theta$  has to vanish.

In the general case, from equation (1.11) it does not follow (explicitly) that the derivatives  $p_i = \partial u / \partial x^i$ , which obey to the equation (and given boundary or initial conditions of the problem), make up a differential. Without any supplementary conditions the commutator  $K_{ij}$  of the differential form  $\theta$  is not equal to zero. The form  $\theta = p_i dx^i$  turns out to be unclosed and is not a differential like the left-hand side of relation (1.12). Functional relation (1.12) appears to be nonidentical.

If to write functional relation (1.12) in the form

$$du - p_i dx^i = 0 \quad (1.12')$$

we get the well-known Pfaff equation for partial differential equation. However, the nonidentical relation cannot be treated as an equation in differentials. To solve equation (1.12') means to find the derivatives  $p_i$  of original equation (1.11) which make up a differential. In this case the derivatives  $p_i$  of equation (1.11) that do not obey these conditions are ignored, although they satisfy original equation (1.11) and boundary and initial conditions. But in the analysis of relation (1.12) all derivatives that satisfy original equation (1.11) and boundary or initial conditions are accounted for, and their role in the physical process under consideration is investigated.

The nonidentity of functional relation means that the original equation is nonintegrable one, that is, the derivatives of this equation are not reduced (without additional condition) to the identical relation of the type  $d\psi = dU$ .

It can be shown that, for every differential equation describing any physical processes, the relevant functional relation constructed of derivatives will be nonidentical ones.

Below a role of such relations in mathematical physics will be disclosed.

While investigating real physical processes one often meets relations that are nonidentical. Usually one treats nonidentical relations like equations, namely, one finds only the values at which nonidentical relation becomes identic. In

doing so, it is not accounted for the fact that the nonidentical relation is often obtained from the description of some physical process and it has physical meaning at every stage of the physical process rather than at the stage when the additional conditions are satisfied.

The approach to nonidentical relation as to a relation rather than to the equation in differentials gives fundamentally new physical results.

**Selfvariation of nonidentical relation.** Nonidentical relation possesses an unique evolutionary property, namely, this relation appears to be a *selfvarying* relation.

The nonidentical relation is selfvarying one since, firstly, it is nonidentical, namely, it contains two objects one of which appears to be unmeasurable (non-invariant), and, secondly, it is an evolutionary relation, namely, the variation of any object of the relation in some process leads to the variation of another object, and, in turn, the variation of the latter leads to variation of the former. Since one of the objects is an unmeasurable quantity, the other cannot be compared with the first one, and hence, the process of mutual variation cannot terminate.

Varying the evolutionary form coefficients leads to varying the first term of the commutator (see Eq.(1.3) for the case of the first-order evolutionary form). In accordance with this variation it varies the second term, that is, the metric form of the manifold varies. Since the metric form commutators specifies the manifold differential characteristics that are connected with the manifold deformation, this points to the manifold deformation. This means that the evolutionary form basis varies. In turn, this leads to variation of the evolutionary form, and the process of intervariation of the evolutionary form and the basis is repeated. The processes of variation of the evolutionary form and the basis are governed by the evolutionary form commutator and it is realized according to the evolutionary relation.

Selfvariation of the evolutionary relation goes on by exchange between the evolutionary form coefficients and the manifold characteristics. This is an exchange between physical quantities and space-time characteristics. (This is an exchange between quantities of different nature).

The process of the evolutionary relation selfvariation cannot come to the end. This is indicated by the fact that both the evolutionary form commutator and the evolutionary relation involve unmeasurable quantities. From this it follows that the evolutionary nonidentical relation cannot be converted into identical relation. However, from this nonidentical relation it can be obtained identical relation.

It appears that this is only possible under degenerate transformation.

### **Degenerate transformations. Obtaining identical relation from nonidentical one**

To obtain the identical relation from the evolutionary nonidentical relation, it is necessary that the closed exterior differential form should be derived from the

evolutionary differential form included into evolutionary relation.

However, as it has been shown above, the evolutionary form cannot be a closed form. For this reason the transition from evolutionary form is possible only to the *inexact* closed exterior form that is defined only on pseudostructure.

To the pseudostructure it is assigned a closed dual form (whose differential vanishes). For this reason the transition from the evolutionary form to the closed inexact exterior form proceeds only when the conditions of vanishing the dual form differential are realized, in other words, when the metric form differential or commutator becomes equal to zero.

The metric form of original manifold, on which the evolutionary form (and correspondingly, nonidentical relation) is defined, is not closed. The transition from unclosed metric form with nonzero differential to closed metric form with the differential being equal to zero is possible only as a degenerate transformation, that is, as a transformation which does not conserve the differential.

The conditions of vanishing the dual form differential (vanishing the metric form commutator) are the conditions of degenerate transformation.

Vanishing the metric form commutator under degenerate transformation points to emergence of closed metric form, and this corresponds to realization of pseudostructure (the dual form).

Vanishing one term of the evolutionary form commutator, namely, the metric form commutator, leads to that the second term of the commutator also vanishes. This is due to the fact that the terms of the evolutionary form commutator correlate between one another. The evolutionary form commutator and, correspondingly, the differential, vanish on pseudostructure, and this means that it appears the unclosed inexact exterior form.

If the conditions of degenerate transformation are realized, from the unclosed evolutionary form, which differential is nonzero  $d\omega^p \neq 0$ , one can obtain a differential form closed on pseudostructure. The differential of this form equals zero. That is, it is realized the transition

$$d\omega^p \neq 0 \rightarrow (\text{degenerate transformation}) \rightarrow d_\pi \omega^p = 0, d_\pi^* \omega^p = 0$$

On the pseudostructure  $\pi$  evolutionary relation (1.10) transforms into the relation

$$d_\pi \psi = \omega_\pi^p \quad (1.13)$$

which proves to be an identical relation. Indeed, since the form  $\omega_\pi^p$  is a closed one, on the pseudostructure this form turns out to be a differential of some differential form. In other words, this form can be written as  $\omega_\pi^p = d_\pi \theta$ . Relation (1.13) is now written as

$$d_\pi \psi = d_\pi \theta$$

There are differentials in the left-hand and right-hand sides of this relation. This means that the relation is an identical one.

Thus, it is evident that under degenerate transformation the identical on pseudostructure relation can be obtained from the evolutionary nonidentical relation. This is due to the realization of closed metric form, and correspondingly to the realization of closed exterior form, and this points to the emergence of pseudostructure and a conservative quantity on pseudostructure. The

pseudostructure with conservative quantity makes up a differential-geometrical structure (the properties of such structure will be defined below).

Under degenerate transformation the evolutionary form differential vanishes only *on pseudostructure*. This is an interior (being equal to zero) differential. The total differential of the evolutionary form is nonzero. The evolutionary form remains to be unclosed, and for this reason the original relation, which contains the evolutionary form, remains to be nonidentical.

Under realization of new additional conditions a new identical relation can be obtained. As a result, the nonidentical evolutionary relation can generate identical relations.

(It can be shown that all identical relations of the exterior differential form theory are obtained from nonidentical relations by applying the degenerate transformation.)

The conditions of degenerate transformation, that is, additional conditions, can be realized, for example, if it will appear any symmetries of the metric form coefficients or their derivatives. *This can happen under selfvariation of the nonidentical relation.*

(While describing material system the conditions of degenerate transformation are connected with degrees of freedom of a material system).

To the conditions of degenerate transformation there corresponds a requirement that some functional expressions become equal to zero. Such functional expressions are Jacobians, determinants, the Poisson brackets, residues, and others.

Mathematically the degenerate transformation is realized as a transition from one frame of reference to another (nonequivalent) frame of reference. This is a transition from the frame of reference connected with the manifold whose metric forms are unclosed to the frame of reference being connected with pseudostructure. The first frame of reference cannot be inertial or locally-inertial frame. The evolutionary form and nonidentical evolutionary relation are defined in the noninertial frame of reference. But the closed exterior form obtained and the identical relation are obtained with respect to the locally-inertial frame of reference.

**Relation between degenerate transformations of exterior forms and nondegenerate transformations of evolutionary forms.** In the theory of closed exterior forms only nondegenerate transformations, which conserve the differential, are used. Nondegenerate transformations of evolutionary forms are transformations that do not conserve the differential. Nevertheless, these transformations are mutually connected. The degenerate transformations execute a transition from original deforming manifold to pseudostructures. And the nondegenerate transformations execute a transition from one pseudostructure to another. As the result of degenerate transformation the closed inexact exterior form arises, and this points to origination of the differential-geometrical structure. But under nondegenerate transformation the transition from one closed form to another takes place, and this points to the transition from one differential-geometrical structure to another. When describing the evolution-



ary processes in material media using evolutionary forms it will be shown that the degenerate transformation describes an origination of physical structures (which are differential-geometrical structures) in material media and that non-degenerate transformation describes a transition from one physical structure to another.

## **Mechanism of realization of conjugated objects and operators.**

The mathematical apparatus of evolutionary differential forms reveals the process of mutual topological variation of the evolutionary form and the manifold. Under such variation the conditions of degenerate transformation can be realized, and the exterior differential form closed on the pseudostructure can arise (spontaneously).

To the closed exterior forms it can be assigned conjugated operators, whereas to the evolutionary forms there correspond nonconjugated operators. The transition from the evolutionary form to the closed exterior form is that from nonconjugated operators to conjugated ones. This is expressed as the transition from the nonzero differential (unclosed evolutionary form) to the differential (closed exterior form) that equals zero.

Here it should be emphasized that the properties of deforming manifolds and skew-symmetric differential forms on such manifolds, namely, evolutionary forms, play a principal role in the process of conjugating.

The process of conjugating includes the following points:

1) selfvariation of nonidentical relation, namely, mutual variations of the evolutionary form coefficients (which have the algebraic nature) and of manifold characteristics (which have the geometric nature) described by nonidentical evolutionary equation, and 2) realization of the degenerate transformation.

Hence one can see that the process of conjugating is a mutual exchange between the quantities of different nature and the degenerate transformation under additional conditions. Here it should be pointed that the condition of degenerate transformation (vanishing some functional expressions like, for example, Jacobians, determinants and so on) may be realized spontaneously while selfvarying the nonidentical relation if any symmetries appear. It is possible if the system (that is described by this relation) possesses any degrees of freedom.

One can see that the process of realization of conjugated operators or objects is described by the nontraditional mathematical apparatus, namely, by nonidentical relations and degenerate transformations.

As it has been shown above, closed exterior forms appear in many mathematical formalisms. Practically all conjugated objects are explicitly or implicitly connected with closed exterior forms. And yet it can be shown that closed exterior forms are generated by evolutionary differential forms, which are skew-symmetric differential forms defined on the deforming varying manifolds.

By comparing the exterior and evolutionary forms one can see that they possess the opposite properties. Yet it was shown that the evolutionary differential

forms generate the closed exterior differential forms. This elucidates that the exterior and evolutionary differential forms are unified whole.

### **1.5 Evolutionary forms: Realization of differential-geometrical structures and forming pseudometric and metric manifolds**

#### **Realization of differential-geometrical structures.**

The process of realization of closed exterior form relates to the realization of closed metric form, which is a form being dual with respect to the closed exterior form. The closed exterior form and corresponding dual form make up a new conjugated object. Since the closed exterior form is a conservative quantity (because the differential of closed exterior form equals zero) and the dual form describes the pseudostructure, such conjugated object (closed exterior form and dual form) describes the differential-geometrical structure, namely, the pseudostructure with conservative quantity. As it has been already noted, such structures are the example of the differential-geometrical G-structures. (The physical structures, which forms physical fields, and corresponding conservation laws, are just such structures).

The mathematical apparatus of evolutionary differential forms describing the process of generation of closed inexact exterior differential forms, describes thereby the process of origination of the differential-geometrical structure.

Obtaining differential-geometrical structures is a process of conjugating the objects. Such process is, firstly, a mutual exchange between the quantities of different nature (for example, between the algebraic and geometric quantities or between the physical and spatial quantities), and, secondly, the establishment of exact correspondence (conjugacy) of these objects. This process is described by selfvariation of nonidentical relation and degenerate transformation.

While describing the process of realization of the differential-geometrical structure the following questions arise. What does generate such structures, and what is the reason of such processes? These problems will be discussed in the next sections while studying physical processes in material media (which are controlled by conservation laws).

Below it is shown by what are defined the characteristics of the structures originated, how are such structures classified, how are formed the pseudometric and metric manifolds from these structures.

#### **Characteristics of the differential-geometrical structures realized.**

For the sake of convenience in the subsequent presentation the differential-geometrical structures, to which inexact exterior and dual forms obtained from the evolutionary forms correspond, will be called the binary structures or Bi-Structures.

Since the closed exterior and dual differential forms, which correspond to Bi-Structure arisen, were obtained from the nonidentical relation that involves

the evolutionary form, it is evident that the characteristics of such structure have to be connected:

- a) with those of the evolutionary form and of the deforming manifold on which this form is defined,
- b) with the values of commutators of the evolutionary form and the manifold metric form, and
- c) with the conditions of degenerate transformation as well.

The condition of degenerate transformation corresponds to a realization of the closed metric (dual) form and defines the pseudostructure.

Vanishing the interior commutator of the evolutionary form (on pseudostructure) corresponds to a realization of the closed (inexact) exterior form and points to emergence of conservative (invariant) quantity.

When Bi-Structure originates, the value of the total commutator of the evolutionary form containing two terms is nonzero. These terms define the following characteristics of Bi-Structures:

- a) the first term of evolutionary form commutator (which is composed of the derivatives of the evolutionary form coefficients) defines the value of the discrete change of conservative quantity, that is, the quantum, which the quantity conserved on the pseudostructure undergoes at the transition from one pseudostructure to another;
- b) the second term (which is composed of the derivatives of coefficients of the metric form connected with the manifold) specifies the characteristics of Bi-Structures, which fixes the character of the initial manifold deformation taking place before Bi-Structures arose. (Spin is such an example). (This characteristics fixes the deformation of original manifold that proceeded in the process of originating the differential-geometrical structure and was described by selfvariation of nonidentical relation).

Thus, the conditions of degenerate transformation determine the pseudostructures; the first term of the evolutionary form commutator determines the value of the discrete change (the quantum) of conservative quantity; the second term of the evolutionary form commutator specifies the characteristics that fixes the character of the initial manifold deformation.

The discrete (quantum) change of conservative quantity proceeds in the direction that is normal to the pseudostructure. (Jumps of the derivatives normal to the potential surfaces are examples of such changes.)

Above it has been noted that the evolutionary form and the nonidentical relation are obtained while describing the physical processes that proceed in material systems. For this reason it is evident that the characteristics of Bi-Structure must also be connected with the characteristics of the material system being described.

**Classification of differential-geometrical structures realized.** The closed forms that correspond to Bi-Structures are generated by the evolutionary relation which includes the evolutionary form of  $p$  degree. Therefore, the structures originated can be classified by the parameter  $p$ .

The other parameter is the degree of closed forms generated by the nonidentical evolutionary relation.

The nonidentical evolutionary relation with the forms of degree  $p$  will be called the evolutionary relation of degree  $p$ . Under degenerate transformation from *nonidentical* relation of degree  $p$  an *identical* relation is obtained. This identical relation contains the closed form of degree  $k = p$ .

To find how the parameter  $k = p$  changes, one has to consider the problem of integration of the nonidentical evolutionary relation.

The identical relation with closed form of degree  $p$  obtained can be integrated, since the right-hand side of such a relation can be expressed in terms of differential (as well as the left-hand side).

By integrating the identical relation obtained one can get the nonidentical relation of degree  $(p - 1)$ .

The nonidentical relation of degree  $(p - 1)$  obtained can be integrated once again if the corresponding degenerate transformation is realized, and the identical relation is formed. This identical relation will already include the closed form of degree  $k = p - 1$ .

In this manner the evolutionary nonidentical relation of degree  $p$  may generate closed (on the pseudostructure) exterior forms of sequential degrees  $k = p, \dots, k = 0$  and corresponding identical relations.

Thus, one can see that Bi-structures, to which there are assigned the closed (on the pseudostructure) exterior forms, can depend on two parameters. These parameters are the degree of evolutionary form  $p$  (in the evolutionary relation) and the degree of created closed forms  $k$ .

In addition to these parameters, another parameter appears, namely, the dimension of space. If the evolutionary relation generates the closed forms of degrees  $k = p, k = p - 1, \dots, k = 0$ , to them there are assigned the pseudostructures of dimensions  $(N - k)$ , where  $N$  is the space dimension.

## Forming pseudometric and metric manifolds.

At this point it should be noted that at every stage of the evolutionary process it is realized only one element of pseudostructure, namely, a certain minipseudostructure. (The example of minipseudostructure is the wave front. The wave front is an eikonal surface (the level surface), i.e. a surface with conservative quantity.)

While varying the evolutionary variable the minipseudostructures form the pseudostructure.

Manifolds with closed metric forms are formed by pseudostructures. They are obtained from the deforming manifolds with unclosed metric forms. In this case the initial deforming manifold (on which the evolutionary form is defined) and the manifold with closed metric forms originated (on which the closed exterior form is defined) are different spatial objects.

It takes place the transition from the initial (deforming) manifold with unclosed metric form to the pseudostructure, namely, to the manifold with closed

metric forms created. Mathematically this transition (the degenerate transformation) proceeds as *a transition from one frame of reference to another, nonequivalent, frame of reference*.

The pseudostructures, on which the closed *inexact* forms are defined, form the pseudomanifolds.

To the transition from pseudomanifolds to metric space it is assigned the transition from closed *inexact* differential forms to *exact* exterior differential forms.

It was shown above that the evolutionary relation of degree  $p$  can generate (with using the degenerate transformations) closed forms of degrees  $0, \dots, p$ . While generating closed forms of sequential degrees  $k = p, k = p - 1, \dots, k = 0$  the pseudostructures of dimensions  $(n + 1 - k)$  are obtained. As a result of transition to the exact closed form of zero degree the metric structure of the dimension  $n + 1$  is obtained.

Sections of the cotangent bundles (Yang-Mills fields), cohomologies by de Rham, singular cohomologies, pseudo-Riemannian and pseudo-Euclidean spaces, and others are examples of the pseudostructures and spaces that are formed by pseudostructures. Euclidean and Riemannian spaces are examples of metric manifolds that are obtained when changing to the exact forms. Here it should be noted that the examples of pseudometric spaces are potential surfaces (surfaces of a simple layer, a double layer and so on). In these cases the type of potential surfaces is connected with the above listed parameters.

Conservative quantities (closed exterior inexact forms) defined on pseudomanifolds (closed dual forms) constitute some fields. (The physical fields are the examples of such fields.) The fields of conservative quantities are formed from closed exterior forms at the same time when the manifolds are created from the pseudostructures.

Since the closed metric form is dual with respect to some closed exterior differential form, the metric forms cannot become closed by themselves, independently of the exterior differential form. This proves that the manifolds with closed metric forms are connected with the closed exterior differential forms. This indicates that the fields of conservative quantities are formed from closed exterior forms at the same time when the manifolds are created from the pseudostructures. The specific feature of manifolds with closed metric forms that have been formed is that they can carry some information. That is, the closed exterior differential forms and manifolds, on which they are defined, are mutually connected objects.

One can see that the evolutionary forms possess the properties, which enable one to describe the evolutionary processes, namely, the processes of generating the differential-geometrical structures and forming manifolds. In other mathematical formalisms there are no such possibilities that the mathematical apparatus of evolutionary and exterior skew-symmetrical forms possesses.

The evolutionary differential forms, which generate the closed exterior forms, disclose the mechanism of origination of physical structures forming physical fields and the determinacy of these processes.

Before describing a role of the evolutionary differential forms in mathematical physics and field theory, it is necessary to show the relation between the closed exterior forms and existing invariant field theories.

## 2 Role of exterior forms in field theory

The role of exterior forms in field theory is due to the fact that they reflect the properties of conservation laws.

### 2.1 Conservation laws for physical fields (exact conservation laws)

Owing to the development of science the concept of “conservation laws” in thermodynamics, physics and mechanics contains different meanings.

In areas of physics related to the field theory and in the theoretical mechanics “the conservation laws” are those according to which there exist conservative physical quantities or objects. These are the conservation laws that above were called “exact”.

In mechanics and physics of continuous media the concept of “conservation laws” is related to the conservation laws for energy, linear momentum, angular momentum, and mass that establish the balance between the change of physical quantities and external action. These are balance conservation laws.

In thermodynamics the conservation laws are associated with the principles of thermodynamics.

Below it will be shown the relation between exact and balance conservation laws and the connection of conservation laws with the principles of thermodynamics.

*The exact conservation laws are those that state an existence of conservative physical quantities or objects. The exact conservation laws are related to physical fields {The physical fields [9] are a special form of the substance, they are carriers of various interactions such as electromagnetic, gravitational, wave, nuclear and other kinds of interactions.}*

The closed exterior differential forms correspond to the exact conservation laws. Indeed, from the closure conditions of the exterior differential form (see formulas (1.4), (1.6), (1.7)) it is evident that the closed exterior differential form is a conservative quantity. In this case the closed inexact exterior differential form and the corresponding dual form describe a conservative object, namely, there is a conservative quantity only on some pseudostructure  $\pi$ . From this one can see that the closed exterior differential form can correspond to the exact conservation law.

The closure conditions for the exterior differential form ( $d_\pi \theta^p = 0$ ) and the dual form ( $d_\pi {}^* \theta^p = 0$ ) are mathematical expressions of the exact conservation law.

It has been pointed above that the pseudostructure (dual form) and the conservative quantity (the closed exterior form) define the differential-geometrical (binary) structure (Bi-structure), which is the example of G-Structure. It is evident that such structure corresponds to the exact conservation law.

It is such structures (pseudostructures with a conservative physical quantity) that correspond to exact conservation law, that are, the physical structures from which physical fields are formed.

Equations for the physical structures ( $d_\pi \theta^p = 0$ ,  $d_\pi * \theta^p = 0$ ) turn out to coincide with the mathematical expression for the exact conservation law.

The mathematical expression for the exact conservation law and its connection with physical fields can be schematically written in the following manner:

$$\begin{cases} d_\pi \theta^p = 0 \\ d_\pi * \theta^p = 0 \end{cases} \mapsto \begin{cases} \theta^p \\ * \theta^p \end{cases} \text{ --- physical structures } \mapsto \text{ physical fields}$$

It should be emphasized that the closed *inexact exterior forms* correspond to the physical structures that form physical fields. The *exact exterior forms* correspond to the material system elements. (About this it will be said below).

It can be shown that the field theories, i.e. the theories that describe physical fields, are based on the invariant and metric properties of the closed exterior differential and dual forms that correspond to exact conservation laws.

## 2.2 Closed exterior differential forms in invariant field theories. (Exact conservation laws and specific features of existing field theories)

The properties of closed exterior differential forms correspond to the conservation laws for physical fields. Therefore, the mathematical principles of the theory of closed exterior differential forms lie at the basis of existing field theories.

The equations that are equations of the existing field theories are those obtained on the basis of the properties of the exterior differential form theory.

The Hamilton formalism is based on the properties of closed exterior and dual forms of the first degree. The corresponding equation of the field has the form:

$$\frac{\partial s}{\partial t} + H(t, q_j, p_j) = 0, \quad \frac{\partial s}{\partial q_j} = p_j \quad (2.1)$$

where  $s$  is the field function (the state function) for the action functional  $S = \int L dt$ . Here  $L(t, q_j, \dot{q}_j)$  is the Lagrange function,  $H$  is the Hamilton function:  $H(t, q_j, p_j) = p_j \dot{q}_j - L$ ,  $p_j = \partial L / \partial \dot{q}_j$ . To this equation it is assigned the closed exterior form of the first degree, which is the Poincare invariant  $ds = -H dt + p_j dq_j$ . The closure conditions for this form and corresponding dual form constitute the Hamiltonian systems:

$$\frac{dq_j}{dt} = \frac{\partial H}{\partial p_j}, \quad \frac{dp_j}{dt} = -\frac{\partial H}{\partial q_j} \quad (2.2)$$

The Schrödinger equation in quantum mechanics is an analog to equation (2.1) (where the conjugated coordinates are changed by operators), and the Heisenberg equation is an analog to the appropriate integral of the equation (2.2) that have the form of canonical relations. The conjugacy of Dirac's *bra*- and *ket*- vectors in quantum mechanics corresponds to the closure condition of the zero degree exterior form [10]. {The duality of closed forms manifests itself in the approaches by Schrödinger and Heisenberg. Whereas the zero degree closed exterior form corresponds to the Schrödinger equation, the closed dual form corresponds to the Heisenberg equation. It can be pointed out that, whereas the equations by Schrödinger and Heisenberg describe the behavior of the potential obtained from the zero degree closed form, Dirac's *bra*- and *ket*-vectors constitute the zero degree closed exterior form itself as the result of conjugacy (vanishing the scalar product)}.

It is evident that the closed exterior and dual forms of zero degree correspond to quantum mechanics.

The properties of closed exterior and dual forms of the second degree lie at the basis of the electromagnetic field equations. The Maxwell equations may be written as  $d\theta^2 = 0$ ,  $d^*\theta^2 = 0$ , where  $\theta^2 = \frac{1}{2}F_{\mu\nu}dx^\mu dx^\nu$  (here  $F_{\mu\nu}$  is the strength tensor).

Closed exterior and dual forms of the third degree correspond to the gravitational field.

From the above said one can see that to each type of physical fields there corresponds a closed exterior form of appropriate degree. (However, to the physical field of given type it can be assigned closed forms of less degree. In particular, to the Einstein equation for gravitational field it corresponds the first degree closed form, although it was pointed out that the type of a field with the third degree closed form corresponds to the gravitational field.)

The connection between field theory and closed exterior differential forms supports the invariance of field theory.

And here it should underline that field theories are based on the properties of closed *inexact* forms. This is explained by the fact that only inexact exterior forms can correspond to the physical structures that form physical fields. The condition that the closed exterior forms, which constitute the basis of field theory equations, are inexact ones reveals in the fact that essentially all existing field theories include a certain elements of noninvariance, i.e. they are based either on functionals that are not identical invariants (such as Lagrangian, action functional, entropy) or on equations (differential, integral, tensor, spinor, matrix and so on) that have no identical invariance (integrability or covariance). Such elements of noninvariance are, for example, the nonzero value of the curvature tensor in Einstein's theory [3], the indeterminacy principle in Heisenberg's theory, the torsion in the theory by Weyl [3], the Lorentz force in electromagnetic theory [11], an absence of general integrability of the Schrödinger equations, the Lagrange function in the variational methods, an absence of the identical integrability of the mathematical physics equations, and that of identical covariance of the tensor equations, and so on. Only if we assume elements of noncovariance, we can obtain closed *inexact* forms that correspond to physical structures.



And yet, the existing field theories are invariant ones because they are provided with additional conditions under which the invariance or covariance requirements have to be satisfied. It is possible to show that these conditions are the closure conditions of exterior or dual forms. Examples of such conditions may be the identity relations: canonical relations in the Schrödinger equations, gauge invariance in electromagnetic theory, commutator relations in the Heisenberg theory, symmetric connectednesses, identity relations by Bianchi in the Einstein theory, cotangent bundles in the Yang-Mills theory, the Hamilton function in the variational methods, the covariance conditions in the tensor methods, etc. The field theory postulates are the expression of such conditions.

The connection between the field theory equations and closed exterior forms shows that to every physical field it is assigned the appropriate degree of closed exterior form. The type of gauge transformations used in field theory depends on the degree of closed exterior differential form.

This shows that it is possible to introduce a classification of physical fields according to the degree of closed exterior form. But within the framework of only exterior differential forms one cannot understand how this classification is explained. (This can be elucidated only by application of evolutionary differential forms.)

The exterior differential forms, whose properties correspond to the conservation laws, constitute the basis of the invariant field theories.

The existing field theories allow to describe the physical fields. However, because these theories are invariant ones they cannot answer the question about the mechanism of originating physical structures that form physical fields. The origination of physical structures and forming physical fields are evolutionary processes, and hence they cannot be described by the invariant field theories. Only evolutionary theory can do this. The theory of exterior and evolutionary forms can be such a theory.

### 3 Role of evolutionary forms in mathematical physics and field theory

The role of evolutionary forms in mathematical physics and field theory (as well as the role of exterior forms) is due to the fact that they reflect the conservation laws. However, these conservation laws are those not for physical fields but for material media. These are balance conservation laws.

#### 3.1 Balance conservation laws

*The balance conservation laws are those that establish the balance between the variation of a physical quantity and the corresponding external action. These are the conservation laws for the material systems (material media).*

The balance conservation laws are the conservation laws for energy, linear momentum, angular momentum, and mass.

In the integral form the balance conservation laws express the following [12]: a change of a physical quantity in an elementary volume over a time interval is counterbalanced by the flux of a certain quantity through the boundary surface and by the action of sources. Under transition to the differential expression the fluxes are changed by divergences.

The equations of the balance conservation laws are differential (or integral) equations that describe a variation of functions corresponding to physical quantities [8, 12 - 14]. If the material system is not a dynamical one (as in the case of a thermodynamic system), the equations of the balance conservation laws can be written in terms of increments of physical quantities and governing variables.

(The specific forms of these equations for thermodynamical and gas dynamical material systems and the systems of charged particles will be presented in the Appendix).

But it appears that, even without knowledge of the concrete form of these equations, with the help of the differential forms one can see specific features of these equations that elucidate the properties of the balance conservation laws. To do so it is necessary to study the conjugacy (consistency) of these equations.

{The necessity of studying the conjugacy of the equations describing any process has a physical meaning. If these equations (or derivatives with respect to different variables) be not conjugated, the solutions to corresponding equations prove to be noninvariant: they are functionals rather than functions. The realization of the conditions (while varying variables), under which the equations become conjugated ones, leads to that the relevant solution becomes invariant. It will be shown below that the transition to the invariant solution, which can be obtained only using evolutionary forms, describes the mechanism of evolutionary transition from one quality to another, which leads to emergence of physical structures}.

Equations are conjugate if they can be contracted into identical relations for the differential, i.e. for a closed form.

Let us analyze the equations that describe the balance conservation laws for energy and linear momentum.

We introduce two frames of reference: the first is an inertial one (this frame of reference is not connected with the material system), and the second is an accompanying one (this system is connected with the manifold built by the trajectories of the material system elements). The energy equation in the inertial frame of reference can be reduced to the form:

$$\frac{D\psi}{Dt} = A \quad (3.1)$$

where  $D/Dt$  is the total derivative with respect to time,  $\psi$  is the functional of the state that specifies the material system,  $A$  is the quantity that depends on specific features of the system and on external energy actions onto the system. {The action functional, entropy, wave function can be regarded as examples of the functional  $\psi$ . Thus, the equation for energy presented in terms of the action functional  $S$  has a similar form:  $DS/Dt = L$ , where  $\psi = S$ ,  $A = L$  is the Lagrange function. In mechanics of continuous media the equation for energy of

an ideal gas can be presented in the form [13]:  $Ds/Dt = 0$ , where  $s$  is entropy. In this case  $\psi = s$ ,  $A = 0$ . It is worth noting that the examples presented show that the action functional and entropy play the same role.}

In the accompanying frame of reference the total derivative with respect to time is transformed into the derivative along the trajectory. Equation (3.1) is now written in the form

$$\frac{\partial\psi}{\partial\xi^1} = A_1 \quad (3.2)$$

here  $\xi^1$  is the coordinate along the trajectory.

In a similar manner, in the accompanying frame of reference the equation for linear momentum appears to be reduced to the equation of the form (see, for example, Appendix)

$$\frac{\partial\psi}{\partial\xi^\nu} = A_\nu, \quad \nu = 2, \dots \quad (3.3)$$

where  $\xi^\nu$  are the coordinates in the direction normal to the trajectory,  $A_\nu$  are the quantities that depend on the specific features of the system and external (with respect to local domain) force actions.

Eqs. (3.2), (3.3) can be convoluted into the relation

$$d\psi = A_\mu d\xi^\mu, \quad (\mu = 1, \nu) \quad (3.4)$$

where  $d\psi$  is the differential expression  $d\psi = (\partial\psi/\partial\xi^\mu)d\xi^\mu$ .

Relation (3.4) can be written as

$$d\psi = \omega \quad (3.5)$$

here  $\omega = A_\mu d\xi^\mu$  is the skew-symmetrical differential form of the first degree.

Since the balance conservation laws are evolutionary ones, the relation obtained is also an evolutionary relation.

Relation (3.5) was obtained from the equation of the balance conservation laws for energy and linear momentum. In this relation the form  $\omega$  is that of the first degree. If the equations of the balance conservation laws for angular momentum be added to the equations for energy and linear momentum, this form in the evolutionary relation will be the form of the second degree. And in combination with the equation of the balance conservation law of mass this form will be the form of degree 3.

Thus, in the general case the evolutionary relation can be written as

$$d\psi = \omega^p \quad (3.6)$$

where the form degree  $p$  takes the values  $p = 0, 1, 2, 3..$  (The evolutionary relation for  $p = 0$  is similar to that in the differential forms, and it was obtained from the interaction of energy and time.)

In relation (3.5) the form  $\psi$  is the form of zero degree. And in relation (3.6) the form  $\psi$  is the form of  $(p - 1)$  degree.

Let us show that *the evolutionary relation obtained from the equation of the balance conservation laws proves to be nonidentical.*

To do so we shall analyze relation (3.5).

In the left-hand side of evolutionary relation (3.5) there is a differential that is a closed form. This form is an invariant object. The right-hand side of relation (3.5) involves the differential form  $\omega$ , that is not an invariant object because in real processes, as it is shown below, this form proves to be unclosed.

For the form to be closed the differential of the form or its commutator must be equal to zero.

Let us consider the commutator of the form  $\omega = A_\mu d\xi^\mu$ . The components of the commutator of such a form can be written as follows:

$$K_{\alpha\beta} = \left( \frac{\partial A_\beta}{\partial \xi^\alpha} - \frac{\partial A_\alpha}{\partial \xi^\beta} \right) \quad (3.7)$$

(here the term connected with the manifold metric form has not yet been taken into account).

The coefficients  $A_\mu$  of the form  $\omega$  have been obtained either from the equation of the balance conservation law for energy or from that for linear momentum. This means that in the first case the coefficients depend on the energetic action and in the second case they depend on the force action. In actual processes energetic and force actions have different nature and appear to be inconsistent. The commutator of the form  $\omega$  constructed from the derivatives of such coefficients is nonzero. This means that the differential of the form  $\omega$  is nonzero as well. Thus, the form  $\omega$  proves to be unclosed and is not a measurable quantity.

This means that the relation (3.5) involves an unmeasurable term. Such a relation cannot be an identical one.

Hence, without the knowledge of particular expression for the form  $\omega$ , one can argue that for actual processes the relation obtained from the equations corresponding to the balance conservation laws proves to be nonidentical.

In similar manner it can be shown that general relation (3.6) is also non-identical.

As it was noted the differential forms  $\omega^p$  and relations (3.6) are defined on accompanying manifold, that is, on the manifold made up by the trajectories of elements of material system. This manifold is deforming one. The metric forms of such manifold cannot be closed. The differential forms defined on such manifold are evolutionary differential forms, which have been described in the fourth section.

The differential forms  $\omega^p$  are evolutionary forms, and relations (3.5) and (3.6) are examples of nonidentical relations for evolutionary forms.

In the Appendix the derivation of nonidentical relations for thermodynamic and gas dynamic systems as well as for the system of charged particles is presented and their brief analysis is given.

Thus, one can see that to the conservation laws for physical fields (exact conservation laws) there correspond the properties of closed inexact forms, whereas to the conservation laws for material media there correspond the properties of evolutionary differential forms.

The nonidentity of the relation obtained from the equations of balance conservation laws means that the equations of balance conservation laws turn out to be nonconjugated (thus, if from the energy equation we obtain the derivative of  $\psi$  in the direction along the trajectory and from the momentum equation we find the derivative of  $\psi$  in the direction normal to the trajectory and then we calculate their mixed derivatives, from the condition that the commutator of the form  $\omega$  is nonzero it follows that the mixed derivatives prove to be noncommutative).

The nonconjugacy of the equations of balance conservation laws reflects the properties of balance conservation laws, namely, their noncommutativity. This property has a governing importance for the evolutionary processes.

### 3.2 Evolutionary process in material medium and origin of physical structures

In this subsection the mechanism of evolutionary processes in material media, which are accompanied by the emergence of physical structure, is described on the basis of the mathematical apparatus of evolutionary differential forms.

The conservation laws are shown to play a governing role in evolutionary processes.

[To emphasize a connection between the mathematical and physical principles, some of the principles listed above will be included into the titles of some subsections. It will be used double titles, namely, those having the physical meaning and those having the mathematical one. The corresponding mathematical principles are presented in brackets.]

#### Nonequilibrium of the material system. (Nonidentity of the evolutionary relation)

It was shown above that the evolutionary relation that was obtained from the balance conservation law equations proves to be nonidentical. This points to the noncommutativity of the balance conservation laws. By analyzing the behavior of the nonidentical evolutionary relation one can understand to what result the noncommutativity of the balance conservation laws leads.

Before we start analyzing the evolutionary relation some relevant concepts such as a “local domain” of a material system, “accompanying manifolds”, “nonequilibrium” and “locally equilibrium” states of a material system are to be explained.

*The local domain of material system.* The local domain of material system is the element and its vicinity. In deriving the evolutionary relation with the first degree form (for the balance conservation laws of energy and linear momentum) it was considered the local domain of material system that involves the material system element and its vicinity, this is, the material system element was an element of the local domain. For the evolutionary relation with the second degree form the local domain for the evolutionary relation with the first degree form will serve as the element of the local domain, and so on. For the evolutionary

relation with zero degree form the element of the material system will serve as the local domain (rather than the element).

*Accompanying manifold.* The accompanying manifold is the manifold constructed by the trajectories of the elements of corresponding local domains. For the evolutionary relation with  $p = 1$  the accompanying manifold is the manifold formed from the trajectories of the material system elements because in this case the elements of the material system in itself serve as the elements of local domain. It is to be noted that the accompanying manifold is constructed of trajectories of the elements of the *local domain*, rather than of *material system* (see the concept of a “local domain”).

*Nonequilibrium and locally equilibrium states of material system.* It is evident that the nonequilibrium is connected with actions of some forces. One has to distinguish between external forces and internal ones (in the subsequent analysis one will also deal with potential forces). Here external forces (external actions) mean the forces that act onto the local domain. Internal forces are those that act inside the local domain of the material system. The nonequilibrium is connected with action of *internal forces*. The nonequilibrium state is the state of a material system when internal forces act in each local domain. If there are no internal forces, the state of material system is in equilibrium. If there are no internal forces only in a particular domain of the material system but there are internal forces in the neighboring local domains, such a state of the material system will be referred to as the state “in local equilibrium”. In this case the total state of the material system is not equilibrium.

We are coming now to the analysis of the evolutionary relation.

It was mentioned above that the noncommutativity of the balance conservation laws is connected with a state of the material system. This is reflected by the evolutionary relation.

It should be emphasized that the evolutionary relation treats a state of the *local domain* of the material system.

{The evolutionary relation takes into account the interaction of an element of the local domain with its vicinity. The noncommutativity of the balance conservation laws points to the fact that the element and its vicinity prove to be nonconjugated.}

Let us consider evolutionary relation (3.5).

If the evolutionary relation proves to be identical, one can obtain the differential  $d\psi$  and find the state function  $\psi$ , this will indicate that the material system state is in equilibrium. But if the evolutionary relation be nonidentical, this indicates an absence of the differential  $d\psi$  and nonequilibrium of the material system state. (Hereafter the differential  $d\psi$  will be called the state differential as it specifies the material system state. This is a closed form. If the state differential be an exact closed form, this corresponds to the equilibrium system state, whereas, if the state differential be an inexact closed form, this will correspond to the locally equilibrium state).

The evolutionary relation gives a possibility to determine either presence or absence of the differential (the closed form). And this allows us, firstly, to rec-

ognize whether the material system state is in equilibrium, in local equilibrium or not in equilibrium, and secondly, to determine the conditions of transition from one state into another (this explains the mechanism of such a transition). If it is possible to determine the differential  $d\psi$  from the evolutionary relation, this indicates that the system is in equilibrium or locally equilibrium state. And if the differential cannot be determined, then this means that the system is in a nonequilibrium state.

It is evident that if the balance conservation laws be commutative, the evolutionary relation would be identical and from that it would be possible to get the differential  $d\psi$ , this would indicate that the material system is in the equilibrium state.

However, as it has been shown, in real processes the balance conservation laws are noncommutative. The evolutionary relation is not identical and from this relation one cannot get the differential  $d\psi$ . This means that the system state is nonequilibrium.

The nonequilibrium state means that there is an internal force in the material system. It is evident that the internal force originates at the expense of some quantity described by the evolutionary form commutator. (If the evolutionary form commutator be zero, the evolutionary relation would be identical, and this would point to the equilibrium state, i.e. the absence of internal forces.) Everything that gives a contribution into the evolutionary form commutator leads to emergence of the internal force.

Thus, the nonidentity of the evolutionary relation obtained from the equations of the balance conservation laws points to the noncommutativity of the balance conservation laws and the nonequilibrium material system state produced as a result. A quantity described by the evolutionary differential form commutator serves as the internal force.

Each external action, as the result of which a change of physical quantities has been produced, has the nature different from that of the material system itself. For this reason the changed physical quantities cannot directly become the physical quantities of the material system itself. (The noncommutativity of the balance conservation laws does not allow a direct transition of the external actions into the physical quantities of the material system). The changed physical quantities prove to be inconsistent. As a result it arises an unmeasurable quantity that is described by the commutator of the evolutionary form  $\omega^p$  and acts as an internal force.

To become the consistent physical quantities of the material system itself, the modified physical quantities have to come to agreement with the properties of the material system. Such transitions, as it will be shown below, are also governed by the balance conservation laws.

### **Selfvariation of nonequilibrium state of material system. (Selfvariation of the evolutionary relation)**

What does the material system nonequilibrium indicated by the nonidentity of the evolutionary relation results in?

While describing the properties of evolutionary forms it has been shown that, the evolutionary and nonidentical relation is a selfvarying one.

Such a specific feature of the evolutionary relation explains the particulars of the material system, namely, *the selfvariation of its nonequilibrium state*.

The selfvariation mechanism of the *nonequilibrium* state of the material system can be understood if analyze the selfvariation of the evolutionary relation go on. For this purpose we have to analyze the topological properties of the evolutionary form commutator.

The evolutionary form in the evolutionary relation is defined on the accompanying manifold that for real processes appears to be the deformable manifold because it is formed simultaneously with a change of the material system state and depends on the physical processes. Such a manifold cannot be a manifold with closed metric forms. Hence, the term containing the characteristics of the manifold will be included into the evolutionary form commutator in addition to the term connected with derivatives of the form coefficients. The interaction between these terms of different nature describes a mutual change of the state of the material system.

Let us examine this with an example of the commutator of the form  $\omega = A_\mu d\xi^\mu$  that is included into evolutionary relation (3.5).

We assume that at the beginning the accompanying manifold was that with the first degree closed metric form. In this case the commutator of the form  $\omega$  can be written as (3.7). If at the next instant any action affects the material system, this commutator turns out to be nonzero. The state of the material system becomes nonequilibrium and it will arise an internal force whose action will lead to a deformation of the accompanying manifold. The metric form commutator of the accompanying manifold, which specifies the deformation, will become nonzero (that is, the metric form of the accompanying manifold will be unclosed). In the commutator of the form  $\omega$  it will appear an additional term, that specifies a deformation of the manifold and is a commutator of the manifold metric form. If it is possible to define the coefficients of connectedness  $\Gamma_{\alpha\beta}^\sigma$  (for a nondifferentiable manifold they are skew-symmetric ones), the form commutator may be written as (1.3), where  $A_\alpha = a_\alpha$ .

The emergence of the second term can only change the commutator and cannot make it zero (because the terms of the commutator have different nature). The further deformation (torsion) of the manifold will go on. This leads to a change of the metric form commutator, produces a change of the evolutionary form and its commutator and so on. Such a process is governed by the nonidentical evolutionary relation and, in turn, produces a change of the evolutionary relation.

The process of selfvariation of the evolutionary relation points to a change of the material system state. But the material system state remains nonequilibrium in this process because the internal forces do not vanish due to the evolutionary form commutator remaining nonzero.

At this point it should be emphasized that such selfvariation of the material system state proceeds under the action of internal (rather than external) forces. That will go on even in the absence of external forces. That is, the selfvariation



of the nonequilibrium state of the material system takes place.

Here it should be noted that in a real physical process the internal forces can be increased (due to the selfvariation of the nonequilibrium state of the material system). This can lead to the development of instability in the material system [15]. {For example, this was pointed out in the works by Prigogine [16]. “The excess entropy” in his works is analogous to the commutator of a nonintegrable form for the thermodynamic system. “Production of excess entropy” leads to the development of instability}.

### **Transition of the material system into a locally equilibrium state. Origination of the physical structures. (Degenerate transformation. Emergence of closed exterior forms)**

Thus, it was shown that in real processes the material system is in a nonequilibrium state (with an internal force). This follows from the analysis of the nonidentical evolutionary relation obtained from the balance conservation law equations.

Now the question arises whether the material system can get rid of the internal force and transfer into the equilibrium state?

The internal force is described by the evolutionary form commutator. But the evolutionary form commutator cannot vanish. This means that the internal force, which is described by the evolutionary form commutator, cannot disappear. That is, the material system cannot be transformed into the equilibrium (without internal forces) state.

However, the material system can change from the nonequilibrium state into the locally equilibrium state. This follows from the evolutionary differential form properties. Under degenerate transformation the identical relation can be obtained from the nonidentical evolutionary relation. That is, from nonidentical relation (3.6) it is obtained the identical on pseudostructure relation

$$d_{\pi}\psi = \omega_{\pi}^p \quad (3.8)$$

where the form  $\omega_{\pi}^p$  is one closed on pseudostructure.

The identical relation obtained from the nonidentical evolutionary relation under degenerate transformation integrates the state differential and the closed inexact exterior form. The availability of the state differential  $d_{\pi}\psi$  indicates that the material system state becomes a locally equilibrium state (that is, the local domain of the system under consideration changes into the equilibrium state). The availability of the exterior closed inexact form  $\omega_{\pi}^p$  means that the physical structure is present. This shows that the transition of material system into the locally equilibrium state is accompanied by the origination of physical structures.

The conditions of degenerate transformation are connected with symmetries that can be obtained from the coefficients of commutators of evolutionary and metric forms. Such symmetries can be due to the degrees of freedom of material system and its elements. The translational degrees of freedom, internal degrees

of freedom of the system elements, and so on can be examples of such degrees of freedom.

As it was noted above, to the degenerate transformation it must correspond vanishing of some functional expressions, such as Jacobians, determinants, the Poisson brackets, residues and others. Vanishing these functional expressions is the closure condition for a dual form. And it should be emphasize once more that *the degenerate transformation is realized as the transition from the accompanying noninertial coordinate system to the locally inertial system.*

The availability of the degrees of freedom in the material system indicates that it is allowed the degenerate transformation, which, in turns, allows the state of the material systems to be transformed from a nonequilibrium state to a locally equilibrium state. But, for this to take place in reality it is necessary that the additional conditions connected with the degrees of freedom of the material system be realized. It is selfvariation of the nonequilibrium state of the material system describe by the selfvarying evolutionary relation that could give rise to realization of the additional conditions. This can appear only spontaneously because it is caused by internal (rather than external) reasons (the degrees of freedom are the characteristics of the system rather than of external actions).

Under degenerate transformation the evolutionary form differential vanishes only along a certain direction. In other words, the interior differential equal to zero is realized. But in this case the total differential of the evolutionary form is nonzero. The evolutionary form commutator does not vanish. Vanishing the interior differential of the evolutionary form means that there is a closed inexact form, and this points to the locally equilibrium state of the material system. At the same time a nonzero value of the total differential of evolutionary form means that the form remains unclosed. This shows that the total state of the material system remains nonequilibrium.

Thus, from the properties of the nonidentical evolutionary relation and those of the evolutionary form one can see that under realization of the additional condition (which is a condition of degenerate transformation) the transition of the material system state from nonequilibrium to locally equilibrium state can be realized. Such transition is accompanied by emergency of physical structures. It should be emphasized once again that such a transition can occur only spontaneously.

Here we should recall once again that the closed inexact form is a quantity with double meaning, namely, it is both the conservative quantity and the measurable quantity that acts as a potential force. The transition of the material system from nonequilibrium into a locally equilibrium state (which is indicated by the formation of a closed form) means that the unmeasurable quantity described by the nonzero commutator of the nonintegrable differential form  $\omega^p$ , that acts as an internal force, transforms into the measurable quantity. It is evident that it is just the measurable quantity that acts as a potential force. In other words, the internal force transforms into a potential force.

Thus, the mathematical apparatus of the evolutionary differential forms elucidates a mechanism of the evolutionary process in material media and of the

emergency of physical structures. This mechanism involves the following steps.

1) The external actions onto the material system are transformed into the unmeasurable quantity that acts as an internal force and brings the material system into the nonequilibrium state. (*The nonzero value of the evolutionary form commutator. Nonidentity of the evolutionary relation obtained from the balance conservation laws*).

2) Selfvariation of the nonequilibrium state of the material system. The deformation of accompanying manifold. (*Selfvariation of the nonidentical evolutionary relation. The topological properties of the evolutionary form commutator*).

3) Realization of the degrees of freedom of the material system in the process of selfvariation of the nonequilibrium state of the system itself. (*Degenerate transformations*).

4) Transition of the material system from the nonequilibrium state into the locally equilibrium one: the transition of an internal force into a potential force. The emergence of physical structures. (*Formation of closed inexact forms and obtaining the differential  $d_\pi\psi$  that specifies the state of the material system*).

{Here it should be emphasized the following.

The evolutionary relation nonidentity that follows from the conservation law noncommutativity just reflects the overdeterminacy of the set of the balance conservation law equations. Actually, a number of the balance conservation law equations is equal to a number of desired physical quantities that specify the material system. But since the physical quantities relate to the same material system, it has to be some connection between them. (This connection is executed by the function that specifies the system state.) And as the physical quantities are related to each other, then the set of the balance conservation law equations proves to be overdetermined one. A realization of the additional conditions, when from the nonidentical evolutionary relation it follows the identical relation, there corresponds to that from the overdetermined set of equations it results the set of consistent equations from which one can find the desired physical quantities. As the additional conditions may be realized only in the discrete manner, then the solutions to this set may be only quantized.

In the book by A.Pais “The Science and the Life of Albert Einstein” the author wrote: “He (Einstein) hoped that the idea of the overdeterminacy will lead to getting the discrete solutions. He also believed that from the future theory it will be possible to derive the partly localized solutions that would correspond to particles that carries the quantized electric charge”.}

## 4 Evolutionary forms: Properties of physical structures. Formation of physical fields and manifolds

In Section 3 the mechanism of evolutionary processes in material media was described, and it was shown that the evolutionary processes lead to origination

of the physical structures. (These are just such structures that form physical fields.)

In the present Section the properties of the physical structures and their connection with material media are described. The mechanism of forming physical fields and corresponding manifolds is described.

#### **4.1 Characteristics of physical structures. (Characteristics of differential forms)**

The exterior form closed on the pseudostructure in combination with the dual form determining the pseudostructure constitute the differential-geometrical structure named the Bi-Structure.

The physical structure is such differential-geometrical structure.

The physical structure is an object obtained by conjugating the conservative physical quantity, which is described by inexact closed exterior form, and the pseudostructure, which is described by relevant dual form.

What characteristics do the physical structures possess?

The closed exterior forms corresponding to physical structures are conservative quantities. These conservative quantities describe certain charges.

Under transition from one structure to another the conservative quantity corresponding to the closed exterior form discretely changes, and the pseudostructure also changes discretely.

Discrete changes of the conservative quantity and pseudostructure are determined by the value of the evolutionary form commutator, which the commutator has at the instant when the physical structure originates. The first term of the evolutionary form commutator obtained from the derivatives of the evolutionary form coefficients controls the discrete change of the conservative quantity. The second one obtained from the derivatives of the metric form coefficients of the initial manifold controls the pseudostructure change.

Spin is the example of the second characteristic. Spin is a characteristic that determines a character of the manifold deformation before origination of the quantum. (The spin value depends on the form degree.)

Breaks of the derivatives of the potential along the direction normal to the potential surface, breaks of the derivative in transition throughout the characteristic surfaces and in transition throughout the wave front, and others are the examples of discrete change of the conservative quantity.

A discrete change of the conservative quantity and that of the pseudostructure produce the quantum that is obtained while going from one structure to another. The evolutionary form commutator formed at the instant of the structure origination determine characteristics of this quantum.

## 4.2 Connection between physical structures originated and material systems. (Identical relation: connection between the state differential and the closed inexact exterior form)

Since closed inexact exterior forms corresponding to physical structure are obtained from the evolutionary relation for the material system, it follows that physical structures are generated by the material systems. (This is controlled by the conservation laws.) The closed exterior forms obtained correspond to the state differential for material system. The differentials of entropy, action, potential and others are the examples of such differentials.

In this manner the physical structures are connected with the material system, its elements, its local domains. The characteristics of physical structure are determined by the characteristics of the material system that generates these physical structures.

The equation of the pseudostructure (dual form) is obtained, as it has been shown, from the condition of degenerate transformation. And this relates to the degrees of freedom of the material system.

The characteristics of inexact closed exterior forms are defined by the characteristics of evolutionary forms following from the balance conservation laws for material media and, hence, depend on the characteristics of material media.

Here it should call attention to one more fact. The material system can generate physical structures only if the system is in nonequilibrium state, that is, if it experience the influence of any actions. As it has been shown, the internal forces arisen are described by the evolutionary form commutator. Therefore, the characteristics of the physical structures arisen will also depend on the characteristics of the evolutionary form commutator describing any actions onto the material system.

In material system the origination of physical structure reveals as a new measurable and observable formation that spontaneously arises in material system. *{As the examples it can be fluctuations, pulsations, waves, vortices, and creating massless particles.}*

In the physical process this formation is spontaneously extracted from the local domain of material system and so it allows the local domain of material system to get rid of an internal force and come into the locally equilibrium state.

The formation created in a local domain of material system (at the cost of unmeasurable quantity that acts in the local domain as an internal force) and liberated from that, begins acting onto the neighboring local domain as a force. This is a potential force, this fact is indicated by the double meaning of the closed exterior form (on the one hand, a conservative quantity, and, on other hand, a potential force). (This action was produced by the material system in itself, and therefore this is a potential action rather than an arbitrary one).

The neighboring domain of the material system works over this action that appears to be external with respect to that. If in the process the conditions of conjugacy of the balance conservation laws turn out to be satisfied again, the

neighboring domain will create a formation by its own, and this formation will be extracted from this domain. (If the conjugacy conditions are not realized, the process is finished.) In such a way the formation can move relative to the material system. (Waves are the example of such motions).

The extraction of a formation from the local domain of the material system is accompanied by emergence of the break surfaces in the material system. The contact breaks are the examples of such surfaces. These breaks do not propagate relative to the material system as shocks or shock waves do. The black holes may be such break surfaces.

The observed formation and the physical structure are not identical objects. If the wave be such a formation, then the wave front is the physical structure. In this case the wave element is a minipseudostructure.

How is the created formation connected with a change of physical quantities of the material system?

Assume that at some instant the local domain of the material system was in equilibrium. That is, its physical quantities, for example, energy and momentum were consistent and simultaneously measurable physical quantities. Then under the effect of external (with respect to the local domain) actions the physical quantities were changed and ceased to be consistent measurable quantities. When the degrees of freedom of the material system are realized, this allows the physical quantities, which were changed at the expense of external actions, to redistribute in such a way as to become measurable physical quantities, namely, the inherent (corresponding to the nature of material system) quantities of material system.

It is evident that the transition from the initial measurable physical quantities to new physical quantities realized is discrete one.

That is, the proper measurable physical quantities are changed discretely. This discrete change of physical quantities is revealed as a formation created.

### **Potential forces. (Duality of closed exterior forms as conservative quantities and as potential forces)**

As it was shown above, the unmeasurable quantity, that acts as an internal force and has been stored at the cost of all external actions giving contribution into the commutator, is converted into a measurable quantity that acts as a potential force. This is indicated by the availability of a closed inexact form that can correspond to the potential force.

Where, from what, and on what does the potential force act?

The potential force is an action of the created (quantum) formation onto the local domains of the material system over which it is translated. And if the internal force acts in the interior of the local domain of the material system (and it caused it to deform), the potential force acts onto the neighboring domain. The local domain gets rid of its internal force and modifies it into a potential force that acts onto the neighboring domains. An unmeasurable quantity, that acts in a local domain as an internal force, is transformed into a measurable quantity of the observable formation (and the physical structure as well) that is

emitted from the local domain and acts onto the neighboring domain as a force equal to this quantity.

The potential forces, as well as the internal forces, originate at the cost of the external actions, but the potential forces (unlike interior ones) are connected with the measurable quantities.

Unlike arbitrary external forces, the potential forces are those that originate at the expense of external actions processed by the material system.

If the external actions equal zero (the evolutionary form commutator be equal to zero), then internal and potential forces equal zero.

Thus, one has to recognize the forces of three types: 1) external forces (external actions), 2) internal forces that originate in local domains of material system due to the fact that the physical quantities of the material system changed by external actions turn out to be inconsistent, and 3) the potential forces are forces of the action of the formations (corresponding to physical structures) onto a material system.

The potential forces are the source of originating internal forces in the neighboring domains of the material system, on which the potential forces act (such as the forces that are external with respect to that domain). For this reason the total state of a material system can remain nonequilibrium just without additional external actions (nonpotential forces).

The potential force, whose value is conditioned by the quantity of the commutator of the evolutionary form  $\omega^p$  at the instant of the formation production, acts normally to the pseudostructure, i.e. with respect to the integrating direction, along which the interior differential (the closed form) is formed. The potential forces are described, for example, by jumps of the derivatives in the direction normal to the characteristics, to the potential surfaces and so on (as well as the physical structure characteristics). This corresponds to the fact that the evolutionary form commutators along these directions are nonzero.

The duality of the closed inexact form as a conservative quantity and as a potential force shows that the potential forces are the action of formations corresponding to the physical structures onto the material system.

Here the following should be pointed out. The physical structures are generated by local domains of the material system. They are the elementary physical structures. By combining with one another they can form the large-scale structures and physical fields.

**Characteristics of the formation created: intensity, vorticity, absolute and relative speeds of propagation of the formation. (Value of the evolutionary form commutator, the properties of material system)**

As it was already mentioned, in the material system a created physical structure is revealed as an observable formation. It is evident that the characteristics of the formation, as well as those of the created physical structure, are determined by the evolutionary form and its commutator and by the material system

characteristics.

Analysis of the formation and its characteristics allows a better understanding of the specific features of physical structures.

Since the formation is a result of converting an unmeasurable quantity described by the evolutionary form commutator into a measurable physical quantity, it is evident that the intensity of the formation created (as well as the discrete change of the physical structure characteristics) is controlled by the quantity that was stored by the evolutionary form commutator at the instant when the formation appeared.

The first term of the commutator constructed of the derivatives of the form coefficients controls the intensity of the formation, whereas the second term that specifies the deformation of the accompanying manifold (bending, torsion, curvature) is fixed as any internal characteristics of the formation originated (which corresponds to vorticity, for example).

In the preceding subsection it was shown that the formation emerged in the local domain of material system acts onto the neighboring local domain of material system. Such action is determined by potential force. It is evident that the formation intensity is revealed as a potential force.

The other characteristics of the formation observed are absolute and relative speeds of the formation propagation.

As it was pointed out above, the observed formation that appears in material system moves along material system. The physical structure is a front of such moving formation.

Here it should be emphasized once again that the moving formation at each instant appears as a newly appeared formation.

The absolute propagation speed of the formation originated (speed in the inertial frame of reference) is obtained from the condition of degenerate transformation that corresponds to conjugacy of the balance conservation laws. As it was already pointed out, these conditions are determined by the material system characteristics and are connected with the degrees of freedom of material system.

These conditions are the differential equations of pseudostructure. They specify the rate of formatting pseudostructure, namely, the speed of the front of the formation emerged. This is just an absolute speed of the formation emerged.

Here it should be emphasized that the speed of the formation originated, though it is determined by the material system characteristics, is not a parameter of the system itself. This is a quantity that at every instant while the formation moves with respect to material system is realized anew as the condition of conjugacy of the balance conservation laws.

If the material system is homogeneous, the speed of translation will have the same values. However, it is not constant because it is formed anew at every instant of the evolutionary processes.

The relative speed (speed in the accompanying frame of reference) is a speed of the formation translation *relative to material system*.

The relative speed is equal to the absolute speed minus the velocity of the local domain elements of material system (or of the elements of material system



if  $p = 1$ ). That is, the relative speed of the observed formation is determined by the degrees of freedom of material system and by the velocities of the elements of local domain.

In such a way the following correspondence between the characteristics of the formations emerged and characteristics of the evolutionary forms, of the evolutionary form commutators and of the material system is established:

1) an intensity of the formation (a potential force)  $\leftrightarrow$  *the value of the first term in the commutator of nonintegrable form* at the instant when the formation is created;

2) vorticity (an analog of spin)  $\leftrightarrow$  *the second term in the commutator that is connected with the metric form commutator*;

3) an absolute speed of propagation of the created formation (the speed in the inertial frame of reference)  $\leftrightarrow$  *additional conditions connected with degrees of freedom of material system*;

4) a speed of the formation propagation relative to material system  $\leftrightarrow$  *additional conditions connected with degrees of freedom of material system and the velocity of elements of local domain*.

Analysis of formations originated in material system that correspond to physical structures enables us to clarify some properties of physical structures.

The rate of varying the pseudostructures that are described by dual forms defines the absolute speed of formations. That is, the pseudostructures are connected with the front of the formation translation. Since a certain physical quantity is conserved on the pseudostructure (the closed form), the pseudostructure is a level surface. The equation of pseudostructure is the equation of eikonal surface. (The eikonal is an example of physical structure.)

It can be shown that the equations of the characteristic surfaces, the surfaces of potential (of simple layer, double layer), the residue equations and so on, obtained from the equations of mathematical physics serve as the equations for pseudostructures. {In the papers [8,17] the connection of equations for one, two, ... eikonals with the equations for characteristics, with the Hamilton equation, and others was shown}.

The mechanism of creating the pseudostructures lies at the basis of forming the pseudometric surfaces and their transition into metric spaces (see the next subsection). It should be pointed out that the eigenvalues and the coupling constants appear as the conjugacy conditions for exterior or dual forms, the numerical constants are the conjugacy conditions for exact forms.

### 4.3 Formation of pseudometric and metric spaces. (Integration of the nonidentical evolutionary relation)

The mechanism of forming pseudometric and metric spaces is connected with the creation of pseudostructures.

In Subsection (1.5) it has been shown that the evolutionary nonidentical relation containing the evolutionary form of degree  $p$  may generate the pseudostructure and the closed (on the pseudostructure) exterior forms of sequential

degrees  $k = p, \dots, k = 0$ . And yet the pseudostructure dimensionality depends on the dimension of space on which the exterior forms are defined.

What is implied by the concept “space”?

While deriving the evolutionary relation two frames of reference were used and, correspondingly, two spatial objects. The first frame of reference is an inertial one, which is connected with the space where material system is situated and is not directly connected with material system. This is an inertial space, it is a metric space. (As it will be shown below, this space is also formed by the material system itself.) The second frame of reference is a proper one, it is connected with the accompanying manifold, which is not a metric manifold.

As it is known, the form degree cannot be greater than the space dimension. Therefore, if the dimension of the inertial space is  $n$ , the maximal degree of the form will be  $n$ . For material system in such a space there work the balance conservation laws that are convoluted into the evolutionary relation of the degree  $p$ . In the real processes practically all conservation laws allowed in the space of a given dimension interact with one another. For given  $n$  the value of  $p$  will be practically a maximal value, that is,  $p = n$ .

Assume that  $n = 2$  and  $p = 2$ . If the form  $\omega^2$  is an unclosed form, the commutator of this form will act in the space of  $n + i = 2 + i$  dimension (here the additional dimension is denoted by an imaginary unit as it does not commute with other dimensions). This means that the deformed accompanying manifold will not be imbedded into the original inertial space of dimension  $n$ . This leads to that the dimension of the space formed increases by one.

It was shown above that the evolutionary relation of degree  $p$  can generate (in the presence of the degenerate transformations) closed forms of the degree  $0 \leq k \leq p$  on the pseudostructures. While generating closed forms of sequential degrees  $k = p, k = p-1, \dots, k = 0$  the pseudostructures of dimensions  $(n+1-k)$ :  $1, \dots, n+1$  are obtained. As a result of transition to the exact closed form of zero degree the metric structure of the dimension  $n + 1$  is obtained. Under influence of the external action (and in the presence of degrees of freedom) the material system can transfer the initial inertial space into the space of the dimension  $n + 1$ . {It is known that the skew-symmetric tensors of the rank  $k$  correspond to closed exterior differential forms, and the pseudotensors of the rank  $(N - k)$ , where  $N$  is the space dimension, correspond to the relevant dual forms. The pseudostructures correspond to such tensors, but on the space formed with the dimension  $n + 1$ . That is,  $N = n + 1$ }.

Under the effect of external actions (and in the presence of degrees of freedom) the material system can convert the initial inertial space of the dimension  $n$  into the space of the dimension  $n + 1$ . Thus, a certain stage of forming the metric space is completed. Every material system has the cycle that includes four stages ( $n = 0, 1, 2, 3$ ). The cycle ends and a new cycle can begin. (This corresponds to one system being embedded into another one). The mechanism of formation of the pseudostructures and the metric structures can explain, in particular, how the internal structure of the elements of material system is formed. (See Subsection 6.1).

So it can be seen that the inertial spaces are not absolute spaces where

actions are developed, they are spaces generated by material systems.

The pseudo-Riemann and pseudo-Euclidean spaces and others can be regarded as examples of pseudostructures and spaces that are formed in a similar manner. The Riemann and Euclidean spaces are the example of metric manifolds obtained in changing to exact forms. Below we present a brief analysis of the space corresponding to gravitational field.

## Space of gravitational field

Material system (medium), which generates gravitational field, is a cosmological system. What can be said about the pseudo-Riemann manifold and Riemann space? The distinctive property of the Riemann manifold is an availability of the curvature. This means that the metric form commutator of the third degree is nonzero. Hence, it does not equal zero the evolutionary form commutator of the third degree  $p = 3$ , which involves into itself the metric form commutator. That is, the evolutionary form that enters into the evolutionary relation is unclosed, and the relation is nonidentical.

When realizing pseudostructures of the dimensions 1, 2, 3, and 4 and obtaining the closed inexact forms of the degrees  $k = 3$ ,  $k = 2$ ,  $k = 1$ , and  $k = 0$ , the pseudo-Riemann space is formed, and the transition to the exact form of zero degree corresponds to the transition to Riemann space.

It is well known that while obtaining the Einstein equations it was suggested that there are fulfilled the conditions [3,18]: the Bianchi identity is satisfied, the coefficients of connectedness are symmetric, the condition that the coefficients of connectedness are the Christoffel symbols, and an existence of the transformation, under which the coefficients of connectedness vanish. These conditions are the conditions of realization of the degenerate transformations for nonidentical relations obtained from the evolutionary relation of the degree  $p = 3$  and after changing to the exact relations. In this case to the Einstein equation there corresponds the identical relations of the first degree.

The above described mechanism of forming the manifolds elucidates the connection between the space and material objects. In his paper [19] S.Weinberg gives more than one historical concepts of the space. He wrote that the connection of the space with material objects was pointed out by Leibnitz who believed that there is no philosophical necessity in any concept of space apart from that following from the connections with material objects. In addition, S.Weinberg cites another similar concept, namely, the Mach principle, which claims that in the definition of inertial system the masses of Earth and celestial bodies play a role. The idea of physical space as a continuum whose properties are governed by the matter was realized by A.Einstein. The above described mechanism of formatting manifolds is one more substantiation of the connection of the space with material objects.

#### 4.4 Forming physical fields. Classification of physical structures. (Parameters of closed and dual forms)

Since the physical structures are generated by numerous local domains of material system and at numerous instants of realizing various degrees of freedom of material system, it is evident that they can generate fields. In this manner physical fields are formed. To obtain the physical structures that form a given physical field, one has to examine the material system corresponding to this field and the appropriate evolutionary relation. In particular, in the Appendix it is shown that, for to obtain the thermodynamical structures (fluctuations, phase transitions, etc), one has to analyze the evolutionary relation for thermodynamical systems, to obtain the gas dynamic ones (waves, jumps, vortices, pulsations) one has to employ the evolutionary relation for gas dynamic systems, for the electromagnetic field one must employ a relation obtained from equations for charged particles.

Closed forms that correspond to physical structures are generated by the evolutionary relation having the parameter  $p$  that defines a number of interacting balance conservation laws. Therefore, the physical structures can be classified by the parameter  $p$ . The other parameter is a degree of closed forms generated by the evolutionary relation. As it was shown above, the evolutionary relation of degree  $p$  can generate closed forms of degree  $0 \leq k \leq p$ . Therefore, physical structures can be classified by the parameter  $k$  as well. Closed exterior forms of the same degree realized in spaces of different dimensions prove to be distinguishable because the dimension of the pseudostructures, on which the closed forms are defined, depends on the space dimension. As a result, the space dimension also specifies the physical structures. This parameter determines the properties of physical structures rather than their type.

Hence, from the analysis of the evolutionary relation one can see that the type and the properties of the differential-geometrical structures and, consequently, of the physical structures (and, accordingly, of physical fields) for a given material system depend on a number of interacting balance conservation laws  $p$ , on the degree of closed forms realized  $k$ , and on the space dimension. By introducing a classification with respect to  $p$ ,  $k$ , and space dimension we can understand an internal connection of various physical fields and interactions. Such a connection will be considered in Subsection (6.1).

### 5 Conservation laws. Symmetries. Causality

#### 5.1 Conservation laws

Here it should be emphasized once again the role of the conservation laws in evolutionary processes and the connection between the conservation laws for material systems and those for physical fields (connection between balance conservation laws and exact conservation laws).

From the description of evolutionary process it was seen that in the evolutionary processes proceeded in material medium the balance conservation laws

for energy, linear momentum, angular momentum, and mass play a controlling and governing role. Such a role of the conservation laws for material systems is due to its peculiarities, that is, they turn out to be noncommutative ones.

This noncommutativity of balance conservation laws reflects the fact that the external actions cannot directly be converted into quantities (measurable) of material system, and hence, they produce a certain nonmeasurable quantity, which acts as internal force and converts the material system into the nonequilibrium state.

The interaction of the noncommutative balance conservation laws controls the process of selfvarying the nonequilibrium state of material system. Such a process can cause the realization of the degrees of freedom of material system (if they are available). The degrees of freedom allow the nonmeasurable quantity, which acts as external force, to be converted into the measurable quantities of the material system itself. The physical structures are the result of realizing the measurable quantities of material system. They appear in material system as the production of new formations. And yet the material system is changed into the locally equilibrium state.

Since the exact conservation laws correspond to physical structures, the origination of physical structure points to a realization of the exact conservation law. From here one can see the connection between the balance and exact conservation laws.

The physical structures that correspond to the exact conservation laws are produced by material system in the evolutionary processes, which are based on the interaction of the noncommutative balance conservation laws.

*Noncommutativity of the balance conservation laws and their controlling role in the evolutionary processes accompanied by emerging physical structures practically have not been taken into account in the explicit form anywhere.* The mathematical apparatus of evolutionary differential forms enables one to take into account and to describe these points.

## 5.2 Symmetries

The exterior and evolutionary skew-symmetric differential forms, which describe the conservation laws, disclose thereby the properties and specific features of symmetries.

The gauge symmetries, which are interior symmetries of the field theory equations, are connected with the conservation laws for physical fields. These symmetries are those of (inexact) closed forms.

The closure property of the exterior form means that any objects, namely, elements of the exterior form, components of elements, elements of the form differential, exterior and dual forms, and others, turn out to be conjugated. And the conjugacy is possible only if there are symmetries of one or other type.

The gauge transformations of field theory, which are nondegenerate transformations of the closed exterior differential forms, are connected with the gauge symmetries. Since the closed exterior differential form is a differential (a total one if the form is exact, or an interior one on pseudostructure if the form

is inexact), it is obvious that the closed form proves to be invariant under all transformations that conserve the differential. The unitary transformations (0-form), the tangent and canonical transformations (1-form), the gradient and gauge transformations (2-form) and so on are examples of such transformations. *These are gauge transformations for spinor, scalar, vector, and tensor (3-form) fields.*

The internal symmetries in field theory are those of closed exterior differential forms, whereas the external symmetries are symmetries of relevant dual forms.

It has been shown that the closed exterior forms and relevant dual forms, which correspond to the conservation laws for physical fields, are obtained from the evolutionary forms, which describe the balance conservation laws for material media. This proceeds under degenerate transformation, which is connected with the degrees of freedom of material system. The conditions of degenerate transformation defines the closed dual form. From this it follows that the external symmetries, namely, the symmetries of dual forms, are due to the degrees of freedom of material system. It is for this reason the exterior symmetries are spatial-temporal symmetries.

The realization of the closed dual form, which proceeds due to the degrees of freedom of material system, leads to realization of the closed exterior form, that is, to the conjugacy of the differential form elements, and emergency of internal symmetries. From this one can see the connection between internal and external symmetries.

Whereas the internal symmetries are connected with the conservation laws for physical fields, the external symmetries caused by the degrees of freedom of material media are connected with the balance conservation laws for material media.

The nondegenerate transformations are connected with internal symmetries, and the degenerate transformations of evolutionary forms are connected with external symmetries.

### 5.3 Causality

The Encyclopedia gives the following definition of the concept of “Causality”. “The causality is a genetic connection between separate states of types and forms of matter in the process of its motion and development. The emergency of any objects and systems and their time-developments have their own grounds in the preceding states of matter; these grounds are known as causes and the changes produced by them are referred to as consequences. The essence of the causality is a production of the consequence by the cause; the consequence, which is governed by the cause, produces the reverse action to the cause”.

A determinacy of the above described evolutionary processes in material systems matches with this definition of the causality.

It can be understood that all external actions onto material system lead to the evolutionary processes. This is a cause of the evolutionary processes.

The emergency of physical structures and formation of physical fields are the consequences of the evolutionary processes in material systems.

From the description of evolutionary processes one can see a connection between physical fields and material systems. Physical fields are generated by material systems. And the noncommutative balance conservation laws for material systems control these processes.

The causality of evolutionary processes in material systems that lead to emergency of physical structures is justified by the mathematical apparatus of evolutionary and exterior differential forms describing the balance conservation laws in material systems.

The evolutionary process in material system and the emergency of physical structures can take place, if

1) the material system is subject to an external action (*the evolutionary form commutator in the evolutionary relation obtained from the balance conservation laws is nonzero*),

2) the material system possesses the degrees of freedom (*there are the conditions of degenerate transformation, under which from the nonidentical evolutionary relation an identical relation is obtained*),

3) the degrees of freedom of material system have to be realized, that is possible only under selfvariations of the nonequilibrium state of material system (*the conditions of degenerate transformation have to be satisfied, this is possible under selfvariation of the nonidentical evolutionary relation*).

If these conditions (causes) are fulfilled, in the material system physical structures arise (*this is indicated by the presence of closed inexact form obtained from the identical relation*).

It is these structures that form physical fields.

Note that to each physical field it is assigned its own material system. The question of what physical system does correspond to a particular physical field is still an open question. Examples of such material systems are the thermodynamic, gas dynamical, cosmological systems, the system of charged particles, and so on. Maybe, for elementary particles the physical vacuum is such a system.

The evolutionary process may lead to creation of elements of its own material system (*while obtaining the exact form of zero degree*).

The emergency of physical structures in the evolutionary process proceeds spontaneously and is manifested as an emergency of certain observable formations. In this manner the causality of emerging various observable formations in material media is explained. Such formations and their manifestations are fluctuations, turbulent pulsations, waves, vortices, creating massless particles, and others.

The existing field theories that are invariant ones are based on some postulates. The investigation performed enables us to make the following conclusion. The postulates, which lie at the basis of the existing field theories, correspond to the conditions of conjugacy of the balance conservation laws for material systems that generate physical structures forming physical fields.

In physics there exists “the causality principle” [9]. This principle establishes the permitted limits of mutual influence of physical events. “The causality principle” is a statement that is significantly narrower than the general philosophic understanding of causality formulated above. It does not clarify the causal and consequential connection of physical phenomena.

## 6 A certain aspects of quantum field theory and approaches to general field theory

The importance of evolutionary forms in mathematical physics and field theory consists in the fact that they disclose the mechanism of forming physical fields and substantiates the determinacy of corresponding processes. From the description of evolutionary processes in material media using the evolutionary skew-symmetric differential forms it follows that material media (material systems) generate physical fields. And the conservation laws, which turn out to be noncommutative, control these processes.

Connection of physical fields with material media and the classification of physical structures based on parameters that specify the conservation laws for material media enable one to see internal relations of different physical fields and their common foundations.

### 6.1 On interactions and classification of physical structures and physical fields

As it was shown above, the type of physical structures (and, accordingly, of physical fields) generated by the evolutionary relation depends on the degree of differential forms  $p$  and  $k$  and on the dimension of original inertial space  $n$  (here  $p$  is the degree of the evolutionary form in the nonidentical relation that is connected with a number of interacting balance conservation laws, and  $k$  is the degree of closed form generated by the nonidentical relation). By introducing the classification by numbers  $p, k, n$  one can understand the internal connection between various physical fields. Since physical fields are the carriers of interactions, such classification enables one to see the connection between interactions.

On the basis of the properties of evolutionary forms that correspond to the conservation laws one can suppose that such a classification may be presented in the form of the table given below. This table corresponds to elementary particles.

{It should be emphasized the following. Here the concept of “interaction” is used in a twofold meaning: an interaction of the balance conservation laws that relates to material systems, and the physical concept of “interaction” that relates to physical fields and reflects the interactions of physical structures, namely, it is connected with the exact conservation laws}.

Recall that the interaction of balance conservation laws for energy and linear momentum corresponds to the value  $p = 1$ , with the balance conservation law



for angular momentum in addition this corresponds to the value  $p = 2$ , and with the balance conservation law for mass in addition it corresponds to the value  $p = 3$ . The value  $p = 0$  corresponds to interaction between time and the balance conservation law for energy.

TABLE					
interaction	$k \setminus p, n$	0	1	2	3
gravitation	3				graviton
					↑
					electron
					proton
electro-magnetic	2			photon2	photon3
				↑	
				electron	
				proton	
weak	1		neutrino1	neutrino2	neutrino3
			↑		
			electron		
			quanta		
strong	0	quanta0	quanta1	quanta2	quanta3
		↑			
		quarks?			
particles					
material	exact	electron	proton	neutron	deuteron?
nucleons?	forms				
N		1	2	3	4
		time	time+	time+	time+
			1 coord.	2 coord.	3 coord.

In the Table names of the particles created are given. Numbers placed near particle names correspond to the space dimension. In braces  $\{\}$  the sources of interactions are presented. In the next to the last row we present massive particles (the elements of material system) formed by interactions (the exact forms of zero degree obtained by sequential integrating the evolutionary relations with the evolutionary forms of degree  $p$  correspond to these particles). In the bottom row the dimension of the *metric* structure created is presented.

From the Table one can see the correspondence between the degree  $k$  of the closed forms realized and the type of interactions. Thus,  $k = 0$  corresponds to the strong interaction,  $k = 1$  corresponds to the weak interaction,  $k = 2$  corresponds to the electromagnetic interaction, and  $k = 3$  corresponds to the gravitational interaction. The degree  $k$  of the closed forms realized and the number of interacting balance conservation laws determine the type of interactions and the type of particles created. The properties of particles are governed by the space dimension. The last property is connected with the fact that closed

forms of equal degrees  $k$ , but obtained from the evolutionary relations acting in spaces of different dimensions  $n$ , are distinctive because they are defined on the pseudostructures of different dimensions (the dimension of the pseudostructure  $(n + 1 - k)$  depends on the dimension of initial space  $n$ ). For this reason the realized physical structures with closed forms of equal degrees  $k$  are distinctive in their properties.

In the Table one cycle of forming physical structures is presented. This cycle involves four levels, to each of which there correspond their own values of  $p$  ( $p = 0, 1, 2, 3$ ) and space dimension  $n$ . The structures formed in the previous cycle serve as a source of interactions for the first level of new cycle. The consequent cycles reflect the properties of material systems subsequently imbedded. And yet a given level has specific properties that are inherent characteristics of the same level in another cycles. This can be seen, for example, from comparison of the cycle described and the cycle in which to the exact forms there correspond conductors, semiconductors, dielectrics, and neutral elements. The properties of the elements of the third level, namely, of neutrons in one cycle and of dielectrics in another are identical to the properties of so called "magnetic monopole" [20,21].

The Table presented provides the idea about the dimension of pseudostructures and metric structures.

In the bottom row of the Table the dimension  $N$  of the metric structure formed is presented. From original space of the dimension 0 the metric space of the dimension 1 (it can occurs to be time) can be realized. From space of the dimension 1 the metric space of the dimension 2 (time and coordinate) can appear and so on. From original space of the dimension 3 it can be formed the metric space of the dimension 4 (time and 3 coordinates). Such space is convoluted and a new dimension cannot already be realized. This corresponds to ending the cycle. (Such metric space with corresponding physical quantity defined by the exact exterior form is the element of new material system.)

The method of studying evolutionary processes developed on the basis of exterior and evolutionary differential forms with using the properties of the conservation laws may serve as an approach to general field theory.

## 6.2 Mathematical apparatus of skew-symmetric differential forms as the basis of general field theory

At Section 2 it was shown the role of closed exterior forms in field theory. The properties of closed exterior differential forms explicitly or implicitly manifest themselves essentially in all formalisms of field theory, such as the Hamilton formalism, tensor approaches, group methods, quantum mechanics equations, the Yang-Mills theory and others.

Such a role of closed exterior forms in field theory is related to the fact that these forms reflect the properties of the conservation laws for physical fields. The differential-geometrical structures described by closed (inexact) exterior forms and relevant dual forms are physical structures that make up physical fields. The degrees of closed exterior forms set the classification of physical

fields and interactions. To the strong, weak, electromagnetic, and gravitational interactions there correspond the closed exterior forms of zeroth, first, second, and third degrees. Gauge transformations for spinor, scalar, vector, and tensor fields are transformations of closed exterior forms of zeroth, first, second, and third degrees.

Such properties of closed exterior forms can be useful in establishing the unified field theory.

And the theory of skew-symmetric differential forms, which unites the theory of closed exterior forms and the theory of evolutionary forms (generating the closed inexact exterior forms), can serve as an approach to the general field theory. Such a theory enables one not only to describe physical fields, but also shows how are the physical fields produced, what does generate them, what is a cause of these processes.

The field theories that are based on exact conservation laws allow to describe physical fields. However, because these theories are invariant ones they cannot answer the question about the mechanism of originating physical structures that form physical fields. The originating physical structures and forming physical fields are evolutionary processes, and hence they cannot be described by the invariant field theories. Only evolutionary theory can do this. The mathematical apparatus of evolutionary differential forms can serve as the mathematical apparatus of such theory.

The basic mathematical foundations that describe the evolutionary process in material systems, and the mechanism of originating physical structures evidently must be included into the evolutionary field theory.

It should be noted that it is rather difficult to realize all these mathematical foundations and in many cases this turns out to be impossible. The difficulties may be caused by a derivation of the evolutionary relation that describes the mechanism of originating physical structures and forming physical fields. To do this one has to know the equations of the balance conservation laws for material systems that generate given physical fields. The problems can be also caused by the fact that these equations have to be written in the frame of reference that is related to the deforming accompanying manifold. Moreover, this can lead to difficulties in the process of obtaining closed exterior forms from evolutionary relation, because for to do this it is necessary to obtain the additional conditions that correspond to the degrees of freedom of material system and have to be realized by themselves when changing the evolutionary relation.

These problems may appear to be practically unsolvable.

However, a knowledge of the basic mathematical principles of evolutionary theory may be helpful while studying the mechanism of originating physical fields.

The results of qualitative investigations of evolutionary processes using the mathematical apparatus of evolutionary differential forms enables one to see the common properties that unify all physical fields. The physical fields are generated by material media, and at the basis of this it lies the interaction of the noncommutative conservation laws of energy, linear momentum, angular momentum, and mass for material media. This explains the causality of physical

phenomena and clarifies the essence of postulates that lie at the basis of existing field theories.

These results enable one to understand what do the properties of physical structures depend on and with which parameters is the classification of physical structures connected, and hence to see the internal connections between various physical fields.

The properties of physical structures depend primarily on which material systems (media) generate physical structures (but the physical structures generated by different material media possess common properties as well).

One of the parameters, according to which it is possible to classify physical structures and physical fields, is the number of interacting balance conservation laws. This is the parameter  $p$  that ranges from 0 to 3. (Recall that the case  $p = 1$  corresponds to interaction of the balance conservation laws of energy and linear momentum, the case  $p = 2$  does to that of energy, linear momentum, and angular momenta, the case  $p = 3$  corresponds to interaction of the balance conservation laws of energy, linear and angular momenta, and mass, and to  $p = 0$  it corresponds an interaction between time and the balance conservation law of energy.) This parameter, which is the evolutionary form degree that enters into the evolutionary relation, specifies a type of physical fields. (So, the electromagnetic field is obtained from interaction between the balance conservation laws of energy and linear and angular momenta. The gravitational field is obtained as the result of interactions between the balance conservation laws of energy, linear momentum, angular momentum, and mass.)

The other parameter is the degree of closed differential forms realized from given evolutionary relation. The values of these parameters denoted by  $\kappa$  range from  $p$  to 0. This parameter, which corresponds to physical structures realized, characterizes the connection between physical structures and exact conservation laws. It specifies a type of interaction of physical fields.

Since the realization of physical structures proceeds discretely, this emphasizes a quantum character of physical fields.

One more parameter is the dimension  $n$  of the space in which the physical structures are generated. This parameter points to the fact that the physical structures, which belong to common type of the exact conservation laws, can be distinguished by their space structure. (The classification with respect to these parameters not only elucidates the connections between the physical fields generated by material media, but explains the mechanism of creating the elements of material media themselves and demonstrates the connections between material media as well).

By comparison of the invariant and evolutionary approaches to field theory one can state the following. Physical fields are described by invariant field theory that is based on exact conservation laws. The properties of closed exterior differential forms lie at the basis of mathematical apparatus of the invariant theory. The mechanism of *forming* physical fields can be described only by evolutionary theory. The evolutionary theory that is based on the balance conservation laws for material systems is just such a theory. It is evident that as the general field theory it must serve a theory that involves the basic mathematical

foundations of the evolutionary and invariant field theories. As an approach to such general field theory it can serve the theory of skew-symmetric differential forms, which unites the theories of closed exterior forms and evolutionary forms.

*The papers on the theory of evolutionary skew-symmetric differential forms written in 2002-2005 years are in ArXive (<http://arXiv.org/find>)*

## Appendix

### The analysis of balance conservation laws for thermodynamic and gas dynamic systems and for the system of charged particles

#### Thermodynamic systems

The thermodynamics is based on the first and second principles of thermodynamics that were introduced as postulates [7]. The first principle of thermodynamics, which can be written in the form

$$dE + dw = \delta Q \quad (A.1)$$

follows from the balance conservation laws for energy and linear momentum (but not only from the conservation law for energy). This is analogous to the evolutionary relation for the thermodynamic system. Since  $\delta Q$  is not a differential, relation (A.1) which corresponds to the first principle of thermodynamics, as well as the evolutionary relation, appears to be a nonidentical relation. This points to a noncommutativity of the balance conservation laws (for energy and linear momentum) and to a nonequilibrium state of the thermodynamic system.

If condition of the integrability be satisfied, from the nonidentical evolutionary relation, which corresponds to the first principle of thermodynamics, it follows an identical relation. It is an identical relation that corresponds to the second principle of thermodynamics.

If  $dw = p dV$ , there is the integrating factor  $\theta$  (a quantity which depends only on the characteristics of the system), where  $1/\theta = pV/R$  is called the temperature  $T$  [7]. In this case the form  $(dE + p dV)/T$  turns out to be a differential (interior) of some quantity that referred to as entropy  $S$ :

$$(dE + p dV)/T = dS \quad (A.2)$$

If the integrating factor  $\theta = 1/T$  has been realized, that is, relation (A.2) proves to be satisfied, from relation (A.1), which corresponds to the first principle of thermodynamics, it follows

$$dS = \delta Q/T \quad (A.3)$$

This is just the second principle of thermodynamics for reversible processes. This takes place when the heat input is the only action onto the system.

If in addition to the heat input the system experiences a certain mechanical action (for example, an influence of boundaries), we obtain

$$dS > \delta Q/T \quad (A.4)$$

that corresponds to the second principle of thermodynamics for irreversible processes.

In the case examined above the differential of entropy (rather than entropy itself) becomes a closed form. {In this case entropy manifests itself as the thermodynamic potential, namely, the function of state. To the pseudostructure there corresponds the state equation that determines the temperature dependence on the thermodynamic variables}.

## Gas dynamical systems

We take the simplest gas dynamical system, namely, a flow of ideal (inviscid, heat nonconductive) gas [13].

Assume that the gas (the element of gas dynamic system) is a thermodynamic system in the state of local equilibrium (whenever the gas dynamic system itself may be in nonequilibrium state), that is, it is satisfied the relation [7]

$$Tds = de + pdV \quad (A.5)$$

where  $T$ ,  $p$  and  $V$  are the temperature, the pressure and the gas volume,  $s$  and  $e$  are entropy and internal energy per unit volume.

Let us introduce two frames of reference: an inertial one that is not connected with material system and an accompanying frame of reference that is connected with the manifold formed by the trajectories of the material system elements.

The equation of the balance conservation law of energy for ideal gas can be written as [13]

$$\frac{Dh}{Dt} - \frac{1}{\rho} \frac{Dp}{Dt} = 0 \quad (A.6)$$

where  $D/Dt$  is the total derivative with respect to time (if to denote the spatial coordinates by  $x_i$  and the velocity components by  $u_i$ ,  $D/Dt = \partial/\partial t + u_i \partial/\partial x_i$ ). Here  $\rho = 1/V$  and  $h$  are respectively the mass and the enthalpy densities of the gas.

Expressing enthalpy in terms of internal energy  $e$  using the formula  $h = e + p/\rho$  and using relation (A.5), the balance conservation law equation (A.6) can be put to the form

$$\frac{Ds}{Dt} = 0 \quad (A.7)$$

And respectively, the equation of the balance conservation law for linear momentum can be presented as [13,22]

$$\text{grad } s = (\text{grad } h_0 + \mathbf{U} \times \text{rot } \mathbf{U} - \mathbf{F} + \partial \mathbf{U} / \partial t) / T \quad (A.8)$$

where  $\mathbf{U}$  is the velocity of the gas particle,  $h_0 = (\mathbf{U} \cdot \mathbf{U})/2 + h$ ,  $\mathbf{F}$  is the mass force. The operator *grad* in this equation is defined only in the plane normal to the trajectory.

Since the total derivative with respect to time is that along the trajectory, in the accompanying frame of reference equations (A.7) and (A.8) take the form:

$$\frac{\partial s}{\partial \xi^1} = 0 \quad (\text{A.9})$$

$$\frac{\partial s}{\partial \xi^\nu} = A_\nu, \quad \nu = 2, \dots \quad (\text{A.10})$$

where  $\xi^1$  is the coordinate along the trajectory,  $\partial s / \partial \xi^\nu$  is the left-hand side of equation (A.8), and  $A_\nu$  is obtained from the right-hand side of relation (A.8).

Equations (A.9) and (A.10) can be convoluted into the equation

$$ds = A_\mu d\xi^\mu \quad (\text{A.11})$$

where  $A_\mu d\xi^\mu = \omega$  is the first degree differential form (here  $A_1 = 0, \mu = 1, \nu$ ).

Relation (A.11) is the evolutionary relation for gas dynamic system (in the case of local thermodynamic equilibrium). Here  $\psi = s$ . {It worth notice that in the evolutionary relation for thermodynamic system the dependence of entropy on thermodynamic variables is investigated (see relation (A.5)), whereas in the evolutionary relation for gas dynamic system the entropy dependence on the space-time variables is considered}.

Relation (A.11) appears to be nonidentical. To make it sure that this is true one must inspect the commutator of the form  $\omega$ .

Nonidentity of the evolutionary relation points to the nonequilibrium state and the development of the gas dynamic instability. Since the nonequilibrium state is produced by internal forces that are described by the commutator of the form  $\omega$ , it becomes evident that the cause of the gas dynamic instability is something that contributes into the commutator of the form  $\omega$ .

One can see (see (A.8)) that the development of instability is caused by not a simply connectedness of the flow domain, nonpotential external (for each local domain of the gas dynamic system) forces, a nonstationarity of the flow.

{In the case when gas is nonideal equation (A.9) can be written in the form

$$\frac{\partial s}{\partial \xi^1} = A_1$$

where  $A_1$  is the expression that depends on the energetic actions (transport phenomena: viscous, heat-conductive). In the case of reacting gas the extra terms connected with the chemical nonequilibrium state are added. These factors contributes to the commutator of the form  $\omega$ .}

All these factors lead to emergency of internal forces, that is, to nonequilibrium state and to development of various types of instability.

And yet for every type of instability one can find the appropriate term giving contribution to the evolutionary form commutator, which is responsible for

this type of instability. Thus, there is an unambiguous connection between the type of instability and the terms that contribute to the evolutionary form commutator in the evolutionary relation. {In the general case one has to consider the evolutionary relations that correspond to the balance conservation laws for angular momentum and mass as well}.

As it was shown above, under realization of additional degrees of freedom it can take place the transition from the nonequilibrium state to the locally equilibrium one, and this process is accompanied by emergency of physical structures. The gas dynamic formations that correspond to these physical structures are shocks, shock waves, turbulent pulsations and so on. Additional degrees of freedom are realized as the condition of the degenerate transformation, namely, vanishing of determinants, Jacobians of transformations, etc. These conditions specify the integral surfaces (pseudostructures): the characteristics (the determinant of coefficients at the normal derivatives vanishes), the singular points (Jacobian is equal to zero), the envelopes of characteristics of the Euler equations and so on. Under crossing throughout the integral surfaces the gas dynamic functions or their derivatives undergo the breaks.

## Electromagnetic field

The system of charged particles is a material medium, which generates electromagnetic field.

If to use the Lorentz force  $\mathbf{F} = \rho(\mathbf{E} + [\mathbf{U} \times \mathbf{H}]/c)$ , the local variation of energy and linear momentum of the charged matter (material system) can be written respectively as [14]:  $\rho(\mathbf{U} \cdot \mathbf{E})$ ,  $\rho(\mathbf{E} + [\mathbf{U} \times \mathbf{H}]/c)$ . Here  $\rho$  is the charge density,  $\mathbf{U}$  is the velocity of charged matter. These variations of energy and linear momentum are caused by energetic and force actions and are equal to values of these actions. If to denote these actions by  $Q^e$ ,  $\mathbf{Q}^i$ , the balance conservation laws can be written as follows:

$$\rho(\mathbf{U} \cdot \mathbf{E}) = Q^e \quad (A.12)$$

$$\rho(\mathbf{E} + [\mathbf{U} \times \mathbf{H}]/c) = \mathbf{Q}^i \quad (A.13)$$

After eliminating the characteristics of material system (the charged matter)  $\rho$  and  $\mathbf{U}$  by using the Maxwell-Lorentz equations [14], the left-hand sides of equations (A.12), (A.13) can be expressed only in terms of the strengths of electromagnetic field, and then one can write equations (A.12), (A.13) as

$$c \operatorname{div} \mathbf{S} = -\frac{\partial}{\partial t} I + Q^e \quad (A.14)$$

$$\frac{1}{c} \frac{\partial}{\partial t} \mathbf{S} = \mathbf{G} + \mathbf{Q}^i \quad (A.15)$$

where  $\mathbf{S} = [\mathbf{E} \times \mathbf{H}]$  is the Poynting vector,  $I = (E^2 + H^2)/c$ ,  $\mathbf{G} = \mathbf{E} \operatorname{div} \mathbf{E} + \operatorname{grad}(\mathbf{E} \cdot \mathbf{E}) - (\mathbf{E} \cdot \operatorname{grad})\mathbf{E} + \operatorname{grad}(\mathbf{H} \cdot \mathbf{H}) - (\mathbf{H} \cdot \operatorname{grad})\mathbf{H}$ .



Equation (A.14) is widely used while describing electromagnetic field and calculating energy and the Pointing vector. But equation (A.15) does not commonly be taken into account. Actually, the Pointing vector  $\mathbf{S}$  must obey two equations that can be convoluted into the *relation*

$$d\mathbf{S} = \omega^2 \quad (\text{A.16})$$

Here  $d\mathbf{S}$  is the state differential being 2-form and the coefficients of the form  $\omega^2$  (the second degree form) are the right-hand sides of equations (A.14) and (A.15). It is just the evolutionary relation for the system of charged particles that generate electromagnetic field.

By analyzing the coefficients of the form  $\omega^2$  (obtained from equations (A.14) and (A.15)), one can assure oneself that the form commutator is nonzero. This means that from relation (A.16) the Pointing vector cannot be found. This points to the fact that there is no such a measurable quantity (a potential).

Under what conditions can the Pointing vector be formed as a measurable quantity?

Let us choose the local coordinates  $l_k$  in such a way that one direction  $l_1$  coincides with the direction of the vector  $\mathbf{S}$ . Because this chosen direction coincides with the direction of the vector  $\mathbf{S} = [\mathbf{E} \times \mathbf{H}]$  and hence is normal to the vectors  $\mathbf{E}$  and  $\mathbf{H}$ , one obtains that  $\text{div}\mathbf{S} = \partial s / \partial l_1$ , where  $S$  is a module of  $\mathbf{S}$ . In addition, the projection of the vector  $\mathbf{G}$  on the chosen direction turns out to be equal to  $-\partial I / \partial l_1$ . As a result, after separating from vector equation (A.15) its projection on the chosen direction equations (A.14) and (A.15) can be written as

$$\frac{\partial S}{\partial l_1} = -\frac{1}{c} \frac{\partial I}{\partial t} + \frac{1}{c} Q^e \quad (\text{A.17})$$

$$\frac{\partial S}{\partial t} = -c \frac{\partial I}{\partial l_1} + c \mathbf{Q}'^i \quad (\text{A.18})$$

$$0 = -\mathbf{G}'' - c \mathbf{Q}''^i$$

Here the prime relates to the direction  $l_1$ , double primes relate to the other directions. Under the condition  $dl_1/dt = c$  from equations (A.17) and (A.18) it is possible to obtain the relation in differential forms

$$\frac{\partial S}{\partial l_1} dl_1 + \frac{\partial S}{\partial t} dt = - \left( \frac{\partial I}{\partial l_1} dl_1 + \frac{\partial I}{\partial t} dt \right) + (Q^e dt + \mathbf{Q}'^i dl_1) \quad (\text{A.19})$$

Because the expression within the second braces in the right-hand side is not a differential (the energetic and force actions have different nature and cannot be conjugated), one can obtain a closed form only if this term vanishes:

$$(Q^e dt + \mathbf{Q}'^i dl_1) = 0 \quad (\text{A.20})$$

that is possible only discretely (rather than identically).

In this case  $dS = 0$ ,  $dI = 0$  and the modulus of the Pointing vector  $S$  proves to be a closed form, i.e. a measurable quantity. The integrating direction (the pseudostructure) will be

$$-\frac{\partial S/\partial t}{\partial S/\partial l_1} = \frac{dl_1}{dt} = c \quad (A.21)$$

The quantity  $I$  is the second dual invariant.

Thus, the constant  $c$  entered into the Maxwell equations is defined as the integrating direction.

## References

1. Cartan E., Les Systemes Differentials Exterieurs et Leurs Application Geometriques. -Paris, Hermann, 1945.
2. Schutz B. F., Geometrical Methods of Mathematical Physics. Cambridge University Press, Cambridge, 1982.
3. Tonnelat M.-A., Les principes de la theorie electromagnetique et la relativite. Masson, Paris, 1959.
4. Encyclopedia of Mathematics. -Moscow, Sov. Encyc., 1979 (in Russian).
5. Bott R., Tu L. W., Differential Forms in Algebraic Topology. Springer, NY, 1982.
6. Wheeler J. A., Neutrino, Gravitation and Geometry. Bologna, 1960.
7. Haywood R. W., Equilibrium Thermodynamics. Wiley Inc. 1980.
8. Fock V. A., Theory of space, time, and gravitation. -Moscow, Tech. Theor. Lit., 1955 (in Russian).
9. Encyclopedic dictionary of the physical sciences. -Moscow, Sov. Encyc., 1984 (in Russian).
10. Dirac P. A. M., The Principles of Quantum Mechanics. Clarendon Press, Oxford, UK, 1958.
11. Pauli W. Theory of Relativity. Pergamon Press, 1958.
12. Dafermos C. M. In "Nonlinear waves". Cornell University Press, Ithaca-London, 1974.
13. Clark J. F., Machesney M., The Dynamics of Real Gases. Butterworths, London, 1964.
14. Tolman R. C., Relativity, Thermodynamics, and Cosmology. Clarendon Press, Oxford, UK, 1969.
15. Petrova L. I., On the problem of development of the flow instability. //Basic Problems of Physics of Shock Waves. Chernogolovka, 1987, V. 1, P. 1, 304-306 (in Russian).
16. Prigogine I., Introduction to Thermodynamics of Irreversible Processes. -C.Thomas, Springfield, 1955.
17. Rumer Yu. B. Investigations on 5-Optics. GITTL, Moscow, 1956 (in Russian).
18. Einstein A. The Meaning of Relativity. Princeton, 1953.

19. Weinberg S., Gravitation and Cosmology. Principles and applications of the general theory of relativity. Wiley & Sons, Inc., N-Y, 1972.
20. Dirac P. A. M., Proc. Roy. Soc., **A133**, 60 (1931).
21. Dirac P. A. M., Phys. Rev., **74**, 817 (1948).
22. Liepman H. W., Roshko A., Elements of Gas Dynamics. John Wiley, New York, 1957